Pigou, Becker and the Regulation of Punishment-Proof Firms

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Abstract: We study the use of fines and inspections to control production activities that create external damages. The model contains a continuum of firms, differing in their compliance costs, so that only high-cost firms evade the regulations. If fines are low, then Pigouvian rules for taxing externalities apply, modified to account for costly inspections.   According to Becker's classic work on crime and punishment, however, these inspection costs can be minimized by raising the fines to very high levels. But by bankrupting firms, high fines are shown to increase the external costs generated by a non-compliant firm's production activities, although they reduce the number of firms that fail to comply with the regulation. We analyze this tradeoff in detail, and obtain some unexpected results about how it should be resolved.

1. Introduction

Economic agents engage in a wide variety of activities that generate external effects. For example, drivers impose congestion costs on others when they use public roads and may endanger others by driving recklessly; homeowners may anger neighbors by listening to loud music or by allowing their property to deteriorate; firms may generate hazardous waste as a byproduct of production or expose their workforce to unnecessary health risks by not talking sufficient care in designing their factories; and banks and other depositary institutions may accumulate the types and quantities of assets that increase the risks of financial crises. Society responds to such situations by attempting to regulating behavior and by punishing those who violate the established rules. Sometimes the behavior is criminalized (it is illegal to dump hazardous waste), while in other instances attempts are made to internalize the external damages (toll roads). In the economics literature there are two classic treatments of the issues that surround such activity, due to Pigou (1920) and Becker (1968), but the analyses differ in focus, and they offer solutions that have starkly different tones. Our goal in this paper is to offer a new approach that unifies the messages of Pigou and Becker by showing that the optimal policy prescription for activities that generate external costs can take on either form, and identifying the conditions that determine which form it takes.

Pigou addressed the issue of externalities in *The Economics of Welfare.* An externality arises whenever the social cost of an activity differs from the private cost. Pigou's solution was to add a set of taxes to the price mechanism that would force individuals to internalize the full social costs. Thus, the Pigouvian solution is to set a tax which equals the marginal damage associated with the activity. If the external cost of the activity is low, the Pigouvian tax will be low; whereas activities that generate large external costs will be subject to large Pigouvian taxes. In this sense, the policy prescription proposed by Pigou is one in which the punishment fits the crime. Although Pigou (1954) acknowledged that there will be informational problems both in designing the optimal tax

scheme and implementing it, the issue of compliance played no role in his analysis. In addition, Pigou's analysis did not emphasize the illegal nature of non-compliance.

In contrast, the illegal nature of non-compliance is at the center of Becker's (1968) analysis of such issues in "Crime and Punishment: An Economic Approach." Becker was interested in the question of how society should go about enforcing laws that criminalize activities that generate external costs. He focused on laws that are enforced by random inspection. The key policy parameters are the probability of detection, adjusted by increasing the rate of inspection, and the level of the fine imposed on those convicted of non-compliance. Becker's goal was to find the optimal policy; the one that minimizes the cost of the illegal activity.[1] He argued that because detection is costly while fines are nearly costless, the fine should be raised all the way up to the full wealth of the perpetrator. This policy enables the regulation to be enforced with a low probability and low cost of detection. It is important to note that in Becker's world, it is optimal to set the fine at a very high level, regardless of the costliness of detection and regardless of the extent of the external cost of the activity. Thus, with Becker's policy prescription, the size of the punishment does not necessarily fit the crime – those found guilty of non-compliance are always driven to the edge of bankruptcy regardless of the extent of the external damage.

It is clear that economists were uncomfortable with the counter-intuitive policy prescription of drastically high fines and low audit rates put forth by Becker. In fact, this finding is sometimes referred to as the "Becker conundrum" because we rarely observe such harsh punishment, even though the argument in its favor is clear and compelling.[2] Since 1968, over 200 articles have been

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[1] Becker recognized the need to correct marginal incentives. In fact, in the early part of his paper, he derived the optimal fine for a fixed inspection rate, showing that in the first-best outcome, the expected fine should be set equal to the harm (as noted by Polinsky and Shavell 2000, this result actually dates back to Bentham 1789). However, Becker's focus was on enforcement. In particular, he argued that the existence of enforcement costs ensures that the marginal conditions that define the first-best outcome will not be satisfied. His solution of a high fine coupled with a low inspection rate was designed to minimize the distortions created by such costs.

[2] In a survey of the literature on enforcement, Polinsky and Shavell (2000) provide a proof that the optimal fine is set at its upper limit when offenders are risk-neutral. Comparing this result with actual practice, they argue for

published on the economics of enforcement, with many targeted at conquering the Becker conundrum.[3] In contrast, the robustness of Pigou's main result is rarely questioned.[4] Extensions have tended to focus on problems with implementation or complications that arise when Pigouvian taxes co-exist with other taxes.[5]

In this paper, we argue that for certain regulations, Becker's analysis is too narrow, in the sense that it does not take into account the full implications of high fines. In particular, when firms must borrow or rent capital to produce, but face regulations that are imperfectly enforced, high fines may distort their choice of inputs and create inefficiencies in factor markets. The reason for this is that if fines are high enough to bankrupt firms, they alter the effective cost of capital that firms face. Bankruptcy eliminates the ability of the fine to depend on the firm's capital usage, since the firm knows that if it is caught evading the regulation, then it will pay all of its assets to the government and investors, regardless of the size of the fine. If the external damages created by the firm's activities depend on its capital usage, then high fines may therefore increase these damages for each non-compliant firm. In addition, the firm's owners will realize that additional capital investment cannot alter the assets available to them in the event of bankruptcy (none). Thus, the marginal cost of capital is reduced by an amount that depends on the probability of detection and punishment.

higher fines. "Substantial enforcement costs could be saved without sacrificing deterrence by reducing enforcement effort and simultaneously raising fines."

[3] For example, harsh fines are not optimal if agents are risk averse (Polinsky and Shavell 1979), because high fines impose an additional risk-bearing cost. In addition, if illegal activities can take on different gradations, it is optimal to impose moderate fines on less serious violations, thereby maintaining sufficient marginal incentives to deter more serious offenses (Sandmo 1981). Other approaches concern the optimal treatment of self-reported violations (Innes 1999), the structure of the criminal justice system (Rubenfeld and Sappington 1987; Malik 1990; Andreoni 1991; and Acemoglu and Verdier 2000), and heterogeneity among offenders (Babchuck and Kaplow 1993).

[4] For important exceptions, see Buchanan (1969), Carlton and Loury (1980, 1986) and Kohn (1986). In addition, as is well known, Coase (1960) argued that when transactions cost are low, Pigouvian taxes will not be needed to reach an efficient outcome. He argued that as long as property rights are well defined, economic agents will be able to agree to the first-best outcome and split the surplus that will be created by eliminating distortionary behavior.

[5] The double dividend literature stresses that in addition to correcting behavior, Pigouvian taxes generate revenue for the government. This creates a secondary benefit by allowing the government to reduce other taxes in the economy that may be creating distortions, but the modern literature has emphasized flaws in this argument (see, for example, Bovenberg and de Mooij 1994, Fullerton and Metcalf 1998, or Fullerton, Leicester and Smith 2010). The problems associated with collecting the information required to implement a Pigouvian tax (for example, measuring the true social cost) were stressed Baumol (1972) and a steady stream of related work has followed.

This consideration reinforces our argument that high fines may increase the external damages created by a firm's production activities.

The costs of these distortions from high fines must be balanced against the gains from using high fines to reduce detection costs while increasing the share of firms that comply with the regulation. Below we develop a model that allows us to investigate how this tradeoff should be resolved. We identify conditions under which it is optimal to enforce some regulations with moderate fines and likely detection—the "Pigouvian approach"—while for others, a "Beckerian approach" is optimal, with fines that not only bankrupt some or all firms, but seize some or all of the assets that are involved in the illegal activity.

The Pigouvian approach survives when there are high external costs related to the firm's capital usage, whereas the Beckerian approach is preferable when there are high external costs related to output, regardless of capital intensities. Perhaps more surprising, the Beckerian approach is also preferable when these output-related external costs are low, provided capital-related external costs are unimportant.

Other results are also potentially surprising. In particular, we find that low unit costs for inspecting and detecting evasion of regulations do not necessarily justify the Pigouvian approach, although this approach tends to rely heavily on inspection activities, to keep fines below levels that would bankrupt firms. The basic insight is that low inspection costs may lower the resource costs involved in maintaining a particular compliance level, but they do not increase the maximum feasible level of compliance in our model.

While intuition might suggest that high fines should be avoided when firms' demands for capital are highly elastic, given the capital distortions described above, we show that the conditions under which it is optimal for fines to bankrupt at least some non-compliant firms do not depend on the capital demand elasticity.

A related result is that there can exist optimal fines and inspection rates that that embody both the Pigouvian and Beckerian approaches, in the sense that some non-compliant firms are bankrupted but fines, but not others, although the only innate differences between firms in our model is differences in compliance costs. This finding does not mean that fines are at "intermediate levels." In fact, we show that if some or all non-compliant firms should be bankrupted by fines, then the fines should seize all of their assets, leaving nothing to investors. Rather, other aspects of the fine and inspection policies are adjusted to induce some firms to risk bankruptcy, such as higher fines on detected evaders who have sufficient assets to remain solvent.

Finally, the Becker conundrum survives in unexpected cases in our model. When external damages are low, we find that it is optimal to set fines high enough to not only bankrupt some firms, but seize all of their assets; that is, "minor nuisances" should be addressed with low inspection costs and very high fines, as a means of saving in inspection costs.

In the next section, we use a simplified version of our model to illustrate how bankruptcy alters firm investment behavior in the presence of regulation and punishment. Our central concern is with the structure of fines and inspection rates, but the government can employ a sales tax to offset potentially undesirable impacts of fines and inspections on equilibrium output. Section 3 describes the optimal policy under the constraint that fines are kept low enough not to bankrupt any firms -- the Pigouvian approach. Section 4 describes the equilibrium when fines do create bankruptcies -- the Beckerian approach -- and Section 5 demonstrates that if the government uses such fines, then they should be set high enough to take all of a bankrupt firm's assets, but not necessarily high enough to bankrupt all firms. Sections 6-8 then provide a detailed exploration of the optimal choice between the two approaches. Examples are provided in Section 9.

2. Framework

Our model consists of a perfectly competitive industry, in which firms finance capital on a competitive market for loans. These firms face a government-imposed regulation of some sort. Our model is set up to allow for a wide variety of regulations, including but not limited to, those that restrict the type and quantity of capital (we provide some examples of appropriate regulatory settings in Section 9). Compliance is costly, and we assume that the cost of compliance varies across firms. In equilibrium, some firms choose to comply with the regulation, whereas other firms operate illegally, risking detection and punishment, by evading the regulation. Neither the government nor potential investors can observe the firm's behavior (or its cost of compliance) without monitoring, so investors cannot condition their investment decisions on the legal status of the firm.[6] The government enforces the regulation by randomly inspecting firms and fining evaders. A firm's capital is observed by the government's auditor, so the fine will be allowed to vary with capital usage in our formal model. But for illustrative purposes, we assume a fixed fine in this section. The government's goal is to set the regulation parameters (the inspection rate, fines and possibly taxes) in a manner that maximizes social welfare.

We assume that each risk-neutral firm produces a single unit of output (x) using two inputs, entrepreneurial activity (e) and capital (k), according to a production function, $q(e,k)$, with neo-classical properties. Capital is provided by investors, who are promised that after all markets clear, they will be repaid the principal of the loan along with interest at rate r. The principal consists of the

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[6] Since the market for loans is competitive, all banks earn zero profits. Borrowers are able to obtain loans from multiple sources, and banks cannot increase profits by undertaking the costly activities needed to ascertain the borrower's production plans. We ignore the effects of corporate and personal taxes on the cost of capital. In the absence of these considerations, the analysis does not depend on whether firms finance capital with equity or debt. In the case of equity financing, r becomes a required return on equity that firms must pay if they are not bankrupt. If firms use both debt and equity at the margin, then the required returns may differ, if bankruptcy reduces payments to debt holders, but not equity holders. We avoid this complication, since it would require that we develop a theory of the firm's financial structure.

unit of capital, which does not depreciate, and the cost of a unit of entrepreneurial activity is normalized at one.

The firm's ability to repay investors will be determined by its choice of inputs, its behavior with respect the law, and the size of the potential punishment. In particular, since entrepreneurial assets are the residual claimants, the firm will have assets of $p + k$ to pay principal, interest and fines, where p denotes the price of the product. If an evading firm chooses an input mix that ties up its liquidity, then the fine is paid first and any remaining assets go to investors. If investors receive less than the principal and interest owed to them, the firm is said to be "bankrupt." Evading firms that leave themselves with more liquidity may be able to pay large fines without bankruptcy.

The firm's input decision is depicted in Figure 1 with the convex curve representing the unit isoquant. For law-abiding firms, the isocost curve is a straight-line with a slope of $-r$ and, as is usual, the firm minimizes costs at the tangency of the two curves. These firms always use an efficient mix of inputs if r equals the social opportunity cost of capital (denoted by r^*). Things are somewhat different for evaders; for them, the slope of the isocost curve will also depend on the regulation parameters. To see this, note that for any given level of the fine, F, there exists a critical level of capital, $k_F \equiv \frac{p-F}{r}$, such that an evader that selects $k \geq k_F$ will be bankrupt by the fine if caught violating the law. This firm will realize that it's effective cost of capital changes at k_F. If the firm selects $k \leq k_F$, then it will carry sufficient liquidity to pay the fine and fully repay investors regardless of circumstances. In this range, the firm's effective cost of capital is the same as it is for a law-abiding firm, r. However, if the firm selects $k_o \geq k_F$, it will fully compensate investors when it successfully evades the law, but it will be able to pay investors only the amount $p + k_o - F$ if fined. If we use π to denote the inspection rate, then the marginal cost of capital for evaders is $(1 - \pi)r$ for $k \geq k_F$. The basic idea is that increasing k a unit, financed with borrowing, provides the firm with another unit of assets to pay back principal, but there are no additional assets to pay interest in the

event the firm is fined; investors receive no interest income with probability π. [7] A higher inspection rate lowers this marginal cost because it increases the probability that the interest on additional investment is effectively paid by the government through reduced fine payments, at no additional cost to the firm. As a result, the isocost curve facing an evader is kinked, with a slope of $-r$ for $k \leq k_F$ and $-(1-\pi)r$ for $k \geq k_F$. Since the kink occurs at k_F, it will never be optimal for the firm to use the level of capital that leaves it exactly bankrupt when fined.

Figure 1 illustrates the case where an evading firm is indifferent between choosing low and high levels of k. In other words, the kinked isocost curve has two tangencies with the isoquant, one on each side of the kink. More generally, when the when the fine is low, the kink occurs at a low value for k, and it is optimal for the firm to operate on the steep portion of the isocost curve, at a point such as S in Figure 1 (S for 'solvent'). However, when the fine is high, the kink occurs at a low value of k, and the firm will operate along the flatter portion of the isocost curve, at a point such as O (O for 'overleveraged'). In other words, a high enough fine lowers the marginal cost of capital from r to $(1-\pi)r$, causing the firm to increase its capital from k_s to k_o, and insuring bankruptcy in the event of an inspection.

Thus, severe fines can significantly increase capital usage, resulting in large social costs, particularly if greater capital usage increases external damages from the firm's production activities. Our formal model allows the government to base its fines on how much capital is employed by the firm, but this is not helpful in the case of fines high enough to bankrupt the firm, because all of the firm's assets are lost, creating a fixed maximum punishment. Nevertheless, our analysis will show that high fines are desirable in a variety of circumstances.

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[7] To be precise, for any given F, the expected cost of producing one unit of output is $e + rk + \pi F$ when $k \leq k_F$ and $e + \pi p + (1-\pi)rk$ when $k \geq k_F$. Thus, the marginal cost of capital is r for $k \leq k_F$ and $(1-\pi)r$ for $k \geq k_F$.

3. Pigouvian Regulation

We are now ready to begin our formal analysis, which we divide into three parts. First, in this section, we confine our attention to situations in which the government finds it optimal to use low or modest fines, so that evading firms are not driven to bankruptcy if caught. In the next two sections, we consider the case of severe fines, and, finally, in Sections 6-8 we compare the two outcomes to find the globally-optimal enforcement mechanism.

Each of our perfectly competitive firms employs entrepreneurial activity e and capital k to produce a unit of output. The regulation both restricts k to some socially-optimal level, k^*, and requires that firms reduce any external costs associated with production. In general these external costs will depend on both the level of k and on the regulations involving the production process (e.g., emission controls in the case of pollution or restrictions on the use of particular financial instruments in the case of financial firms). But to simplify the analysis, we assume that firms that comply with the regulation produce no external costs, whereas those who evade the regulation generate $\beta k + \eta$ units of "external activity". If the total output of the private good produced by non-compliant firms is y_n, then total external output is $x_n \equiv (\beta k + \eta)y_n$, which generates an external cost equal to $h(x_n)$, where h is strictly convex. In other words, external costs are allowed to depend on both capital usage and output.

Firms are identical in all aspects except one, the cost of compliance. We use α to denote a firm's cost of complying with the regulation, and we assume that this firm-specific parameter is drawn *after the firm enters the market* from a continuous distribution function, denoted by $G(\alpha)$. Since a complier, or "legal firm," generates no external costs, it is always socially optimal for this firm to choose its capital and entrepreneurial inputs to minimize costs at the social opportunity cost of capital, denoted r^*. Letting c_ℓ^* denote this minimized costs, the total cost of production and compliance is $c_\ell^* + \alpha$ for a legal firm with compliance cost α.

Evaders choose their own capital levels and do not incur compliance costs, but they risk detection and punishment. The probability of detection is the inspection rate, π, which is the same for all firms. The total fine depends linearly on the firm's capital, $tk + T$, defined over all k that are low enough for the fine not to bankrupt the firm. We will later argue that it is desirable to depart from this linear structure once k reaches the level at which the linear fine and capital payments exhaust all of the firm's assets. Thus, the expected total cost for an evader firm is $c_n(r + \pi t) + \pi T$, where r is the interest rate that investors charge the firm, and the cost of capital now includes the expected marginal fine on capital (the sub-script n indicates that this is the cost function for a non-compliant firm). We assume that investors obtain capital at the economy-wide rate (opportunity cost) of r^*. In the Pigouvian equilibrium, firms that evade the regulation choose to carry enough liquidity to repay investors fully, in which case investors charge all firms the interest rate $r = r^*$.

A firm that is indifferent between complying and not complying with the regulation has a compliance cost, $\bar{\alpha}$, that satisfies

(1) $\bar{\alpha} = \pi T + c_n(r^* + \pi t) - c_\ell^*$

All firms with $\alpha \leq \bar{\alpha}$ prefer to operate legally; and all firms with $\alpha \geq \bar{\alpha}$ prefer to evade the regulation.

To complete the model, we now describe the timing of decisions. In the initial stage, ex ante identical firms decide whether to enter the market. In stage two, α is revealed and firms make their input and compliance decisions. In particular, they decide on the mix of entrepreneurial and capital inputs, and obtain funding from investors. These two stages may be viewed as occurring towards the beginning of a period; investors must wait until the beginning of the next period to be paid. During the period, the capital is put in place. The remaining stages then occur towards the beginning of the next period. In stage 3, output is produced and sold, using the capital and entrepreneurial inputs, and the regulatory authority randomly inspects firms, detects non-compliance, and assesses fines, which

must be paid immediately. In stage 4, investors are paid and entrepreneurs receive any remaining assets. The crucial assumption here is that the government collects fines *before* investors are paid. If the fine is set at a high level, then there may not be sufficient assets available to repay the investors if the firm is detected cheating.

We solve the model by backwards induction. The solution the firm's compliance decision is as determined by (1). For the entry decision, since the firms do not know their value of α before entry, their expected profits from production are given by

(2) $E\Pi(p) = \int_0^{\bar{\alpha}}[p - c_\ell^* - \alpha]dG(\alpha) + \int_{\bar{\alpha}}^{\alpha^m}[p - c_n(r + \pi t) - \pi T]dG(\alpha) - \sigma,$

where p is the price of the product, σ is a sunk cost of entry, and α^m is the maximum value of α among firms that produce.[8] For any given set of enforcement parameters (T, t and π), there is a unique value of p at which expected profits are zero. For all higher p, all firms enter and there will be excess supply in the product market; for all lower p, no firm produces. Solving $E\Pi(p) = 0$ for p and using (1) yields the market-clearing price:

(3) $p = c_\ell^* + \bar{\alpha}[1 - G(\bar{\alpha})] + \int_0^{\bar{\alpha}} \alpha dG(\alpha) + \sigma.$

We assume that the government also collects revenue from consumers by imposing a sales tax of τ on this good, so that the price paid by consumers for each unit is $q \equiv p + \tau$. The assumption here is that while some firms evade the regulation, all firms pay the tax. For example, a regulation concerning a production process may be evadable, while no good possibilities exist for selling the product without paying a sales tax.[9]

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[8] For simplicity, we assume that α^m is low enough so that all firms that enter the market choose to produce when they learn their α. Setting fixed cost sufficiently high will raise the equilibrium price p to insure that this assumption holds, given any α^m. We may then also assume that the maximum feasible punishment never causes all firms to comply.

[9] For analyses of the welfare effects of activities undertaken to evade taxes, see Davidson et al. (2005, 2007).

On the demand side of the product market, the representative consumer has the following quasi-linear utility function:

(4) $U(x, q) = I - q + v(x) - h(x_n),$

where I denotes the consumer's lump-sum income and x is total output.[10] Income I consists of an endowment of the numeraire good, plus a government transfer financed by tax revenue and fines. The consumer treats x_n as fixed and chooses x to maximize utility. Thus, x satisfies the following first order condition,[11]

(5) $v'(x) = q = p + \tau$

Summarizing the product market, the producer price of output, p, is determined by the free-entry condition and is given by (3). Total output, x, is determined by the sales tax τ and the solution to the consumer's maximization problem, given by (5). Since each firm produces one unit of output, x also denotes the number of firms with $y_n = 1 - G(\bar{a})$ of these firms evading the regulation.

If evaders choose a relatively low level of capital (so that they carry enough liquidity to fully repay investors in all cases), their expected costs are $c_n(r^* + \pi t) + \pi T$, as previously discussed. But a higher level of capital (as depicted by point O in Figure 1) results in expected total costs of $c_n[(1 - \pi)r^*] + \pi p$, since the fine bankrupts the firm.[12] As described in the previous section, the higher level of capital entails a lower effective cost of capital and leads to a lower payment by the firm when caught evading the regulation.   It is important to note that the expected marginal fine on capital, πt, no longer enters the cost of capital.   Evaders who are not inspected pay no fine, and evaders who are inspected surrender all of their assets to the government and investors. A rise in k

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[10] Production by law-abiding firms creates no external costs because these firms comply with the regulation.
[11] We assume that I is large enough that (5) is satisfied for all relevant q.
[12] At point O in Figure 1, the firm pays e_o to entrepreneurs, $(1 + r)k_o$ to capital owners, including principal, when not inspected (which occurs with probability $1 - \pi$) and $k_o + p - F$ to capital owners, when inspected (which occurs with probability π). Thus, expected production costs at O (excluding principal) are $e_o + (1 - \pi)rk_o + \pi(p - F)$. In addition, the firm faces an expected fine of πF. Summing to get total expected costs, we obtain $e_o + (1 - \pi)rk_o + \pi p = c_n[(1 - \pi)r] + \pi p$.

may increase the amount owed to the government, but the firm does not care about the split of its assets between investors and the government; costs would not change if the government were given all of the firm's assets, leaving investors with none.

For the lower level of capital to be optimal for the firm, as required for a Pigou equilibrium, it must lead to lower or the same expected costs, which occurs when

(6) $\quad \pi(p - T) \geq c_n(r^* + \pi t) - c_n[(1 - \pi)r^*]$

Thus, bankruptcy will not occur in equilibrium if (6) is satisfied by the government's chosen regulation parameters. We refer to (6) as the "Pigou constraint."

We now turn to the government's problem of optimal enforcement. In addition to the external cost of $h(x_n)$, the government must also be concerned about the resources that it devotes to enforcement. This cost is given by $p_a \pi x$, where p_a denotes the cost of inspecting one firm and πx is the total number of inspections that are carried out. Social welfare (W) is given by

(7) $\quad W = v(x) - h(x_n) - x\big[c_\ell^* G(\bar{\alpha}) + c_n^*(r + \pi t)\big(1 - G(\bar{\alpha})\big)\big] + \int_0^{\bar{\alpha}} \alpha dG(\alpha) + p_a \pi + \sigma\big]$

where $x_n = [\beta k(r + \pi t) + \eta]x[1 - G(\bar{\alpha})]$ and $c_n^*(r + \pi t) = e(r + \pi t) + r^* k(r + \pi t)$; that is, the asterisk indicates that we are evaluating the evader's profit-maximizing inputs at the social opportunity cost of capital [r^* for legal firms and $r^* + h'(x_n)\beta$) for evaders]. We assume that lump-sum transfers are available to balance the government budget. Using (1) and (3), we may rewrite the Pigou constraint, given by (6), as follows:

(8) $\quad \pi\{c_\ell^* + \bar{\alpha}[1 - G(\bar{\alpha})] + \int_0^{\bar{\alpha}} \alpha dG(\alpha) + \sigma\} - \bar{\alpha} \geq c_\ell^* - c_n[(1 - \pi)r^*].$

The government's problem is to select the policy variables π, T, t and τ to maximize social welfare, subject to the Pigou constraint and the market equilibrium conditions. But the equilibrium conditions have already been used to state the problem as the maximization of (7), subject to (8). Note first that the fine T does not appear in the problem. Rather, it is replaced with the marginal

compliance cost, $\bar{\alpha}$, as a control variable. Second, the sales tax is replaced by output x. Thus, the control variables are $\pi, t, \bar{\alpha}$, and x. After solving for their optimal values, we can return to the equilibrium conditions and find the values of T and τ that support the equilibrium.

Maximizing (7) over x yields the following first-order-condition

(9) $\quad v'(x) - h'(x_n)(\beta k_s + \eta)(1 - \bar{G}) - [c_\ell^* \bar{G} + c_n^*(1 - \bar{G}) + \int_0^{\bar{\alpha}} \alpha dG(\alpha) + p_a\pi + \sigma] = 0$

where, to shorten notation, we have defined $\bar{G} = G(\bar{\alpha})$, $c_n^* = c_n^*(r + \pi t)$, and $k_s = k(r^* + \pi t)$ with the sub-script s used to reflects that the firm is solvent at this level of k. If we use (5) to substitute for $v'(x)$, (3) to substitute for p, and then solve for τ, we obtain

(10) $\quad \tau = [1 - \bar{G}][h'(x_n)(\beta k_s + \eta) - \bar{\alpha}] + p_a\pi.$

The tax is positive for two reasons. First, when another firm produces, expected inspections rise, with an expected cost equal to $p_a\pi$. Second, the additional firm generates an expected external cost equal to $[1 - \bar{G}]h'(x_n)(\beta k_s + \eta)$, where $1 - \bar{G}$ is the probability that the entrant will fail to comply with the regulation. But there is a decline in expected total compliance costs equal to $[1 - \bar{G}]\bar{\alpha}$. We show below that the excess of this external cost over the reduced compliance cost is positive when inspections are costly. Hence, a positive sales tax is needed to internalize this excess external cost, plus the additional inspection cost.

Note next that the marginal fine on capital, t, enters only the objective function. Thus, we may differentiate the objective function with respect to t, and obtain the following first-order condition:

(11) $\quad t\pi = h'(x_n)\beta$

This is the usual Pigouvian rule: the tax on another unit of an externality-producing activity should equal the marginal external cost from that activity. Here the activity is additional investment. Note that inspection costs do not alter the rule, because t can be adjusted without altering the total number

of inspections or the number of firms that choose to evade the regulation, simply by offsetting any change in t with a change in T, which is the component of the fine that is independent of the firm's capital. In fact, if the optimal total fine, $tk_s + T$, is low, then it may be necessary for T to be negative to support the optimality conditions for both $\bar{\alpha}$ and t.

The remaining control variables are $\bar{\alpha}$ and π, which relate to our central concern: what are the relative uses of fines and inspections in optimal punishments?   Since inspections are costly, the government will clearly want to minimize their use, but faces the Pigou constraint (eq. 8). Thus, this constraint holds with equality, and we can use it to define the inspection rate as a function of the marginal compliance cost, $\pi(\bar{\alpha})$, thereby eliminating the constraint from the optimization problem and leaving $\bar{\alpha}$ as the remaining control variable. Implicit differentiation of the Pigou constraint, noting that the term in curly brackets is p, yields

(12) $\quad \frac{d\pi}{d\bar{\alpha}} = \frac{1-\pi(1-\bar{G})}{p-r^*k_o}$,

where $k_o \equiv k[(1 - \pi)r^*]$, with the sub-script o reflecting that the firm is overleveraged at this k.

Differentiating objective function (7) with respect to $\bar{\alpha}$ now gives the first-order condition

(13) $\quad [h'(x_n)(\beta k_s + \eta) - \bar{\alpha}]\bar{g} = p_a \frac{d\pi}{d\bar{\alpha}}$,

where $\bar{g} \equiv G'(\bar{\alpha})$. As previously described, the term in the square brackets is the net external benefit of additional compliance, recognizing that when another firm complies with the regulation, total compliance cost rise by $\bar{\alpha}$. If we use a higher fine T to increase the marginal compliance rate a unit, then $\bar{g}$ additional firms comply, producing the marginal net external benefit on the left side of (13). But to do so while still satisfying the Pigou constraint, we must raise the inspection probability by $d\pi/d\bar{\alpha}$, generating a marginal cost of $p_a (d\pi/d\bar{\alpha})$. At the optimum, the marginal benefit equals the marginal cost. This equality is illustrated in Figure 2, where the horizontal axis measures the

level of compliance. Note that if external damages are large, the government will want to increase the severity of its policy to deter non-compliance.

To understand the determinants of $d\pi/d\bar{\alpha}$, note that with a binding Pigou constraint (8), $p = \bar{\alpha} + \{c_\ell^* - c_n[(1-\pi)r^*]\}/\pi$. Thus, (12) may be rewritten as follows:

(14) $\quad \dfrac{d\pi}{d\bar{\alpha}} = \dfrac{\pi[1-\pi(1-\bar{G})]}{\bar{\alpha} - \{c_n^*[(1-\pi)r^*] - c_\ell^*\}}$,

where once again an asterisk on the evaders cost function indicates that its profit-maximizing inputs are being evaluated at the social opportunity cost of capital. The expression in the curly brackets is the excess of this social cost over then minimized cost of production, evaluated at r^*, which is the same cost paid by legal firms, c_ℓ^*. This is the usual definition of deadweight loss from a tax or subsidy distortion, but in the present case, it does not include the external costs associated with additional k. We assume that a firm's capital demand goes to infinity as the cost of capital goes to zero, in which case this deadweight loss goes to infinity as π goes to one. This implies that there will be a maximum feasible $\bar{\alpha}$, denoted $\bar{\alpha}^{max}$, at which $d\pi/d\bar{\alpha}$ goes to infinity. We assume that $G(\bar{\alpha}^{max}) < 1$; that is, it is not feasible to obtain complete compliance as a Pigou equilibrium.

To obtain more insight into the determination of $d\pi/d\bar{\alpha}$, we may approximate the cost difference in (14) by using the usual quadratic loss formula:

(15) $\quad c_n^*[(1-\pi)r^*] - c_\ell^* = \dfrac{1}{2}\pi r^*[k_o - k_\ell] = \dfrac{1}{2}\pi^2 r^* k_\ell \varepsilon,$

where $k_\ell = k(r^*)$ and ε is the elasticity of demand for capital, evaluated at r^*. This approximation becomes exact when the capital demand curve is linear. Substituting (15) into (14) yields

(16) $\quad \dfrac{d\pi}{d\bar{\alpha}} = \dfrac{\pi[1-\pi(1-\bar{G}))]}{\bar{\alpha} - \frac{1}{2}\pi^2 r^* k_\ell \varepsilon}$

The presence of the capital elasticity in (16) can explained by noting from the binding Pigou constraint (eq. 8 with an equality) that when $\bar{\alpha}$ rises by some marginal amount, the required rise in π,

$d\pi$, lowers production costs $c_n[(1-\pi)r^*]$ by $k_s r^* d\pi$ for overleveraged firms, which by itself makes the overleverage option more attractive and therefore makes a given marginal rise in π less effective in restoring indifference about becoming overleveraged. The larger fine and inspection rate alter the overleveraged firm's choice of capital, given by $k_s - k_\ell$, the more π must be raised.

The value of $\pi(\bar{\alpha})$ is determined by the differential equation given by (14), once initial conditions are specified. We know that $\pi(0) = 0$, but this alone does not determine $d\pi(\bar{\alpha})/d\bar{\alpha}$ because the numerator and denominator in (16) are both zero at $\bar{\alpha} = 0$. Rather, we can use (8) to find an expression for the limiting value of $\bar{\alpha}$ as π goes to zero: $\bar{\alpha} = \pi[e(r^*) + \sigma]$. Substituting this expression into (16) and taking the limit as π goes to zero gives

(17) $\qquad \dfrac{d\pi(0)}{d\bar{\alpha}} = \dfrac{1}{e(r^*)+\sigma}.$

Thus, a rise in $e(r^*) + \sigma$ lowers the initial value of $d\pi/d\bar{\alpha}$, presumably leading to lower inspection costs at positive values of $\bar{\alpha}$; that is, Pigouvian regulation becomes more attractive. The basic idea is that the higher are fixed costs and entrepreneurial returns, the higher is the equilibrium price of output, and this higher price enables a given rate of compliance to be maintained with a higher fine and lower inspection rate.[13]

4. Becker Equilibria

We now consider enforcement policies that bankrupt at least some inspected evaders. Fines are high enough to bankrupt such firms when Pigou constraint (6) is reversed; that is,

(18) $\qquad \pi(p-T) \leq c_n(r+\pi t) - c_n[(1-\pi)r],$

where r may now exceed r^* to compensate investors for the possibility of bankruptcy. We refer to (18) as the "Becker constraint." When it holds with a *strict* inequality, the two types of evaders,

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[13] The analysis could be generalized without altering the results by assuming that the initial owners of the firm own some amount of the capital. In this case, the value of this capital at r^* would be added to the denominator of (17) and higher fines would be possible in a Pigou equilibrium.

"overleveraged" and "solvent" (when fined) will *always* use different amounts of capital to minimize their costs of production. In particular, as described in Section 2, overleveraged firms will use a higher level of capital, because they realize that if they fined, the marginal capital will be costless.

Equilibria where at least some evaders are overleveraged are referred to as "Becker equilibria." We can also distinguish between partial and full Becker equilibria, depending on whether some or all evaders are overleveraged. Partial Becker equilibria are possible when the Becker constraint holds with equality, in which case $\gamma < 1$ denotes the positive fraction of firms that are overleveraged. To summarize, $\gamma = 1$ in a full Becker equilibrium, $\gamma \in (0,1)$ in a partial Becker equilibrium, and $\gamma = 0$ in a Pigou equilibrium.

Note that an equality on the Becker constraint does not distinguish a partial Becker equilibrium from a full Becker equilibrium. If (18) holds with a strict inequality, the resulting absence of any solvent evaders (when fined) implies that lowering their fine through a reduction in T has no real effects. Thus, any Becker equilibrium can be supported with a fine structure that leaves firms indifferent about becoming overleveraged. This indifference may be illustrated in Figure 1. Once capital equals the cutoff level k_F, above which the firm is bankrupt, the only restriction on where we set the fine at higher k is that it must be high enough to continue to bankrupt the firm. Given market prices, a bankrupt firm does not care if the fine is increased, since the higher fine is financed by reduced payments of interest or principal. For this reason, the Becker constraint does not contain a fine for bankrupt firms. However, we next show that the specific value of the fine on bankrupt firms still matters, because it affects the equilibrium interest rate.

When the government inspects an overleveraged evader, it will now lay claim to some income owed investors in an attempt to collect the unpaid fines. These anticipated seizures will distort the capital market and lead to a higher price of capital for the regulated market. In equilibrium, the profits earned by investors from supplying capital to this industry must exactly offset

losses associated with the expected seizures. The government inspects a particular firm with probability π and seizes $F_o - (p - rk)$ units of assets from that firm if it has not complied with the regulation, where F_o is the total fine. Since the government observes whether the firm is overleveraged once it inspects the firm, F may differ from the total fine that it would collect on an evader that is not overleveraged. Since the fraction of firms that evade the regulation is $1 - \bar{G}$ and γ is the fraction of evaders that are overleveraged, it follows that expected seizures are $\pi\gamma[1 - \bar{G}]\{F_o - p + rk_o\}$. All law-abiding firms employ the socially-optimal capital level, $k_\ell = k(r^*)$, a fraction $(1 - \gamma)$ of all evaders employ $k_s = k(r + \pi t)$ units of capital, and a fraction γ employ $k_o = k[(1 - \pi)r]$ units. Thus, since the investors pay r^* for the capital, their expected profits from supplying capital to this industry at rate r are given by $(r - r^*)\{k_\ell\bar{G} + (1 - \bar{G})[(1 - \gamma)k_s + \gamma k_o]\}$ in the absence of seizures. The equilibrium r is determined by the requirement that these expected profits equal expected seizures:

(19) $(r - r^*)\{\bar{G}k_\ell + (1 - \bar{G})[(1 - \gamma)k_s + \gamma k_o]\} = \pi\gamma(1 - \bar{G})\{F_o - p + rk_o\},$

noting that the expression in the curly brackets is the excess of the fine on overleveraged firms over the difference between total assets, $p + k_o$, and money owed to investors, $(1 + r)k_o$. Since the right-hand-side of (19) is positive in a Becker equilibrium, it must be the case that $r > r^*$ in any such equilibrium. Thus, capital is paid a premium in the regulated industry.

This excess of r over r^* is a major difference between a Pigou equilibrium and a Becker equilibrium. Note, however, that since r depends on the total fine, F, the fine structure for any firms that are not overleveraged may be adjusted without causing changes in r. We may therefore differentiate the objective function with respect to the marginal fine on capital to obtain a modified Pigou rule for the expected marginal fine on the capital employed by the solvent evaders existing in a partial Becker equilibrium:

(20) $t\pi = h'(x_n)\beta - (r - r^*).$

This rule says that the expected fine should be reduced by the excess of r over r^* to offset any investment distortions from the higher interest rate. In other words, the government has the tools to maintain efficient investment incentives for evaders who are not overleveraged. But such tools do not exist for overleveraged firms, since the fine becomes lump-sum in the case of bankruptcy. This is a major shortcoming of large fines.

Consider now the determination of the marginal compliance cost. Although legal firms are required to use the socially-efficient capital, k_ℓ, they now must pay r for this capital, so their costs rise to the level, $c_\ell(r) \equiv e(r^*) + rk(r^*)$, which exceeds c_ℓ^* by $(r - r^*)k(r^*)$. In a Becker equilibrium, the expected cost for overleveraged evaders is $\pi p + c_n[(1 - \pi)r]$. The marginal compliance cost equates this cost with the cost for a legal firm:

(21) $\bar{\alpha} = \pi p + c_n[(1 - \pi)r] - c_\ell(r)$.

Turning to the product market, we can use this equality between costs for marginal legal and evader firms and write the equilibrium price in a form that is similar to (3), modified to reflect the higher interest rate:

(22) $p = c_\ell(r) + \bar{\alpha}[1 - \bar{G}] + \int_0^{\bar{\alpha}} \alpha dG(\alpha) + \sigma$.

Substituting (22) into (21) gives the following fundamental condition for the determination of the marginal compliance cost:

(23) $\pi\{c_\ell(r) + \bar{\alpha}(1 - \bar{G}) + \int_0^{\bar{\alpha}} \alpha dG(\alpha) + \sigma\} - \bar{\alpha} = c_\ell(r) - c_n[(1 - \pi)r]$.

Finally, output and the number of firms are determined, as in the previous section, by the demand side of the product market – in particular, (5).

Figure 3 illustrates the set of inspection rates (π) and marginal compliance costs ($\bar{\alpha}$) under which the economy is in a Becker equilibrium. The Pigou frontier gives the maximum marginal compliance cost $\bar{\alpha}$ that satisfies the Pigou constraint at each inspection rate π. It is obtained by

inverting the previously-derived function, $\pi(\bar{\alpha})$. For the Becker frontier, we drop the Pigou constraint and maximize $\bar{\alpha}$ over all possible fine policies at each π. Becker equilibria lie to the right of the Pigou frontier and include the Becker frontier, but no points to the right of it. We will later see that it is *not* true the points on the Becker frontier must be full Becker equilibria, where all evaders are overleveraged.

Social welfare in a Becker equilibrium is given by a form similar to (7), modified to reflect the social costs for overleveraged firms, $c_n^*((1 - \pi)r)$, and the higher interest rate r faced by all evaders:

(24) $\quad W = v(x) - h(x_n) - x[C^*(r, \bar{\alpha}) + \int_0^{\bar{\alpha}} \alpha dG(\alpha)] - p_a \pi x;$

where $C^*(r, \bar{\alpha}) \equiv c_\ell^* \bar{G} + (1 - \bar{G})\{(1 - \gamma)c_n^*(r + \pi t) + \gamma c_n^*[(1 - \pi)r]\}$ is the expected social cost of production (that is, the cost of capital is evaluated r^* for legal firms and $r^* + h'(x_n)\beta$ for evaders). The only difference between this welfare expression and the welfare expression for a Pigou equilibrium is that overleveraged firms now employ an inefficient input mix.

Moving from the Pigou optimum to a Becker equilibrium enables the government to increase compliance either beyond its Pigou maximum, or to a level that would be obtainable as a Pigou equilibrium but can now be obtained with lower inspection costs. The problem is that fine on overleveraged firms becomes lump-sum, eliminating the government's ability to control the capital used by overleveraged firms. This capital usage is now $k_o = k[(1 - \pi)r^*]$, whereas the efficient capital level would be $k_s = k[r^* + h'(x_n)\beta]$. This inefficient use of capital raises the social cost of production, creating a deadweight loss:

(25) $\quad L = \{e_o + [r^* + h'(x_n)\beta]k_o\} - c_n(r^* + h'(x_n)\beta) = \frac{1}{2}[r^* + \beta h' - (1 - \pi)r]^2 k' \; ,$

where the second equality uses the quadratic loss expression for deadweight loss. This deadweight loss represents the extra social cost involved in a move to a Becker equilibrium.

5. Optimal Fines in Becker Equilibria

Before investigating the desirability of moving to a Becker equilibrium, we investigate fines in this type of equilibrium. In particular, we find that positive inspection costs are not sufficient to overturn Becker's conclusion that the fine should be maximized:

Proposition 1: *Given any inspection rate π and marginal compliance cost $\bar{\alpha}$ that are supported by a Becker equilibrium, welfare is maximized by setting the fine on overleveraged firms at its maximum level, where the government takes all of the firm's assets, leaving nothing for investors.*

Proof: Suppose that the fine is less than its maximum level for overleveraged firms. Recall that we can adjust the fine on solvent evaders to make them indifferent about becoming overleveraged. Then we can raise the fine while lowering the share of evaders that are overleveraged, in a way that keeps the equilibrium r fixed. With r fixed, there is no change in capital demands by overleveraged firms, and so no change in deadweight loss per overleveraged firm; and there is no change in the compliance level. But the fall in the share of evaders that are overleveraged implies lower total deadweight loss, if overleveraged firms generate positive deadweight losses. In this case, welfare rises. If deadweight losses are zero, there is no change in welfare. Q.E.D.

Thus, even for small policy changes from the Pigou optimum to a Becker equilibrium, it will be optimal to raise the fine on some evaders -- the overleveraged firms -- to its highest possible level.

6. Is a Becker Equilibrium Better than the Pigou Optimum?

This section identifies conditions under which welfare can be improved by moving to a Becker equilibrium. In particular, we consider small policy changes from the Pigou optimum that create overleveraged firms, and we ask whether these changes increase welfare.

The introduction of overleveraged firms raises both γ and r, from $\gamma = 0$ and $r = r^*$. But the change in r does not directly cause any marginal distortions, because all existing firms in the Pigou equilibrium are choosing their socially-optimal capital levels. Rather, only the increase in γ matters, and it lowers welfare because each new overleveraged firm is creating the deadweight loss, L, by using too much capital. The marginal welfare change per unit of output x is

(26) $\qquad \frac{dW}{x} = (h'(x_n)(\beta k_s + \eta) - \bar{\alpha})\bar{g}d\bar{\alpha} - p_a d\pi - (1 - \bar{G})Ld\gamma.$

The first and second terms also apply to the Pigou equilibrium and would be equated to zero if the changes in π and $\bar{\alpha}$ were required to satisfy the binding Pigou constraint. But now $\bar{\alpha}$ can be increased with a lower increase in costly inspections. However, raising $\bar{\alpha}$ with fewer additional inspections means a greater increase in fine, which generates the movement of some firms to the overleveraged status. The third term in (26) gives the resulting welfare loss. Given our starting point, $k_\mu = \bar{G}k_\ell + (1 - \bar{G})k_s$ now denotes the average capital used by firms at the Pigou optimum. Assuming that the capital demand curve is linear over the relevant region, which allows us to employ the quadratic deadweight loss approximation in (26), we obtain:

Proposition 2: *Starting from the Pigou optimum, a small change in the fine and inspection rate that causes some firms to become overleveraged is desirable (undesirable) if*

(27) $\qquad \dfrac{\frac{r^* k_\mu}{2(1+r^*)k_o}\left(1+\frac{\pi}{1-\pi}\bar{G}\right)\left(\frac{h'(x_n)\beta}{r^*\pi}+1\right)^2}{[h'(x_n)(\beta k_s + \eta) - \bar{\alpha}]\bar{g}} < (>)1.$

Proof: Start with the optimal $(\pi, \bar{\alpha})$, determined by (12) and (13):

(28) $\qquad h'(x_n)(\beta k_s + \eta)\bar{g} = \bar{\alpha}\bar{g} + p_a \frac{d\pi}{d\bar{\alpha}}; \qquad \frac{d\pi}{d\bar{\alpha}} = \frac{1-\pi(1-\bar{G})}{p-r^*k_o}.$

Next, implement a perturbation in the inspection rate and fine that involves increasing $\bar{\alpha}$ a marginal unit, but with a rise in π that is an amount δ less than the amount needed to remain in the Pigou regime:

(29) $$\frac{d\pi}{d\bar{\alpha}} = \frac{\pi[1-\pi(1-\bar{G})]}{\bar{\alpha} - \{c_n^*[(1-\pi)r^*]-c_\ell^*\}} - \delta.$$

With evaders indifferent about becoming overleveraged in the partial Becker equilibrium, the Becker constraint (18) holds with equality, and (21) can be used to rewrite it in the same form as Pigou constraint (8), modified to reflect a possible $r > r^*$:

(30) $$\pi\{c_\ell(r) + \bar{\alpha}(1-\bar{G}) + \int_0^{\bar{\alpha}} \alpha dG(\alpha) + \sigma\} - \bar{\alpha} = c_\ell(r) - c_n[(1-\pi)r].$$

We may differentiate (30) to find the change in r from r^* needed to keep evaders indifferent about becoming overleveraged, following the changes in the expected fine and inspection rate satisfying (29):

(31) $$\frac{dr}{d\bar{\alpha}} = \frac{\delta(p-rk_o)}{(k_o-k_\ell)(1-\pi)}.$$

Differentiating the condition for capital market equilibrium, given by (19), we obtain the marginal effect of a rise in r from r^* on the fraction of firms that choose to become overleveraged:

(32) $$\frac{d\gamma}{dr} = \frac{k_\mu}{\pi(1-\bar{G})[F_o-p+r^*k_o]}.$$

Multiplying (31) and (32) together then gives

(33) $$\frac{d\gamma}{d\bar{\alpha}} = \frac{\delta(p-rk_o)k_\mu}{(k_o-k_\ell)(1-\pi)\pi(1-\bar{G})[F_o-p+r^*k_o]}.$$

It is clear from the welfare expression (26) that the government will wish to minimize $d\gamma/d\bar{\alpha}$ by choosing the highest possible value of F_o. This is the value that takes all of the firm's assets, $p+k_o$ (recall Proposition 1). In this case, (33) becomes

(34) $\qquad \dfrac{d\gamma}{d\bar{\alpha}} = \dfrac{\delta(p - rk_o)k_\mu}{(k_o - k_\ell)(1-\pi)\pi(1-\bar{G})(1+r^*)k_o}.$

At the Pigou optimum, we know that the welfare change given by (28) equals zero when the change in the inspection rate satisfies (29) with $\delta = 0$. In this case, there is no change in γ, since we are moving along the binding Pigou constraint. Thus reducing $d\pi/d\bar{\alpha}$ by a positive δ, thereby moving into a partial Becker equilibrium, as described by (34), allows us to rewrite the welfare change in (26) as

(35) $\qquad p_a\delta - \dfrac{d\gamma}{d\alpha^*}L(1-\bar{G}) = p_a\delta - \dfrac{\delta(p-rk_o)k_\mu}{(k_o-k_\ell)(1-\pi)\pi(1+r^*)k_o}L.$

Substituting for the numerator from the optimality conditions (28), and using (25) and $k_o - k_\ell = \pi r^* k'$, we then find that welfare rises (falls) if

(36) $\qquad \delta p_a - \dfrac{\delta p_a[1 - \pi(1-\bar{G})]k_\mu[h'(x_n)\beta + \pi r^*]^2}{2[h'(x_n)(\beta k_s + \eta) - \bar{\alpha}]\bar{g}(1-\pi)\pi^2 r^*(1+r^*)k_o} > (<) \ 0$

Rearranging (36) proves (27). $\qquad$ Q.E.D.

We have previously described the term in the denominator of (27) as the net external benefit from additional compliance, where additional compliance is measured by increasing $\bar{\alpha}$ a marginal unit. Let us denote this marginal benefit by $MB_{\bar{\alpha}}$. In a first-best economy with no inspection costs, $MB_{\bar{\alpha}}$ would equal zero. Here, inspection costs imply that it is positive at the Pigou optimum, as determined by first-order condition (13). Thus, $MB_{\bar{\alpha}}$ measures the extent to which the Pigou constraint binds.

On the other hand, the numerator of (27) is related to the costs involved in moving into the partial Becker region. As $\bar{\alpha}$ increases, the market interest rate r rises to support the higher $\bar{\alpha}$ by increasing the effective punishment of detected evaders who are overleveraged. But r must rise a lot if the inspection rate is close to zero, because then $k_o - k_\ell$, is small, indicating that the rise in r does not "punish" evaders much more than legal firms. For this reason, $d\gamma/d\bar{\alpha}$ is inversely proportional

to $k_o - k_\ell$, and therefore inversely proportional to πr^* in the case of a linear capital demand function. By itself, this reasoning suggests that allowing firms to be overleveraged is too socially costly in terms of increased deadweight loss, if initial compliance and inspection rates are low. However, low inspection rates imply low deadweight loss unless β is high. The following proposition resolves this tradeoff:

Proposition 3*: For any $p_a > 0$ and $\beta > 0$, there exists a marginal compliance cost, $\bar{\alpha}' > 0$, such that if the economy is initially on the Pigou frontier at a positive $\bar{\alpha} < \bar{\alpha}'$, then any marginal policy change that creates overleveraged firms causes welfare to fall. But if $\beta = 0$, then any policy change raises welfare, if p_a is sufficiently high, regardless of the initial $\bar{\alpha}$.*

Proof: Equation (17) and optimality condition (28) show that the denominator of (27) converges to a positive number as $\bar{\alpha}$ goes to zero, provided p_a is positive, as assumed. For $\beta > 0$, the numerator goes to infinity as π goes to zero, which happens as $\bar{\alpha}$ goes to zero. Thus, the left side of (27) exceeds the right side for sufficiently small $\bar{\alpha}$, implying that welfare falls.

For the second part of the proposition, note that under our assumption that capital demand goes to infinity as the cost of capital goes to zero, (14) tells us that there is a maximum $\bar{\alpha}$ that satisfies the Pigou constraint with equality, at which $\pi < 1$. Otherwise, the denominator of (14) would become negative, implying that greater compliance can be obtained with a lower inspection rate. Thus, the numerator of (28) stays bounded below some positive number when $\beta = 0$. But by (17) and (28), the denominator increases without bound as p_a rises. It follows that the left side of (27) will fall short of the right side if p_a is sufficiently high.   Q.E.D.

Thus, if the Pigou optimum involves sufficiently low levels of compliance and capital-related external costs are positive ($\beta > 0$), then small policy changes that create overleveraged firms must lower welfare. Pigou optimum will have low levels of compliance if external costs are low or unit

inspecton costs are high. In these cases, Proposition 3 tell us that it is not desirable to create a small number of overleveraged firms. When we consider policy changes that create greater numbers of overleveraged firms, welfare improvements will be possible with low compliance levels and positive values of β, but it will still be necessary to restrict the relative size of β.

Suppose next that external costs are high enough to move the Pigou-optimal marginal compliance cost close to its maximum feasible level, denoted by $\bar{\alpha}^{max}$. In this case, welfare improvements from the Pigou optimum are also possible. [14]

Proposition 4: *For any $p_a >0$ and $\beta \geq 0$, there exists a marginal compliance cost, $\bar{\alpha}$' $< \bar{\alpha}^{max}$, such that if external cost parameter η implies a Pigou optimum with marginal compliance cost between $\bar{\alpha}$' and $\bar{\alpha}^{max}$, then any marginal policy change from this optimum that creates overleveraged firms must raise welfare.*

Proof: Recall that $\pi(\bar{\alpha}^{max}) < 1$. Since $d\pi/d\bar{\alpha}$ goes to infinity as $\bar{\alpha}$ goes to $\bar{\alpha}^{max}$ from below, (28) shows that the marginal benefit term in the denominator of (27) also goes to infinity. But the numerator of (27) stays bounded below some positive number. Thus, the expression in (27) goes to zero, proving the proposition. Q.E.D.

Thus, creating some overleveraged firms is desirable if the external cost component associated with output is high, since then there is a large benefit from being able to raise compliance through the use of high fines. But restrictions must be placed on the relative size of the external cost component that depends on capital, because the inability of fines to influence the capital usage of overleveraged firms becomes increasingly costly as this component rises.

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[14] There are no similar results for a sufficiently low p_a because reducing p_a enough will produce a corner solution, where there is complete compliance at the Pigou optimum, and therefore no scope for increasing compliance. Moreover, it can be shown in this case there are no marginal changes in the fine and inspection rate that increase welfare by lowering compliance.

7. Efficient Becker Equilibria

We now show that in some cases there will exist Becker equilibria in which overleveraged firms create no deadweight losses. The simple idea here is that the equilibrium r can increase so much that it offsets the negative impact of the inspection rate in the cost of capital formula, leaving deadweight losses equal to zero, as defined in (25). Whether this is possible will require that the external costs associated with capital (β) be sufficiently small, since their presence raises the social cost of capital. Also, there cannot be too few non-compliant firms, because then r need not increase much to compensate investors for their bankruptcy risks. The result may be stated as follows:

Proposition 5: For sufficiently small values of the external cost parameter β, *there exists an interval,* $(\bar{\alpha}_1(\beta), \bar{\alpha}_2(\beta)]$, *with* $\bar{\alpha}_1(\beta) < \bar{\alpha}_2(\beta)$, *such that any marginal compliance cost* $\bar{\alpha}$ *in this interval is supported by an efficient Becker equilibrium (L = 0). As β goes to zero,* $\bar{\alpha}_1(\beta)$ *converges to zero and* $\bar{\alpha}_2(\beta)$ *converges to a positive number. A full Becker equilibrium supports* $\bar{\alpha}_2(\beta)$; *that is, all firms are overleveraged.*

Proof: From capital-market condition (19), we may write:

$$(37) \qquad r^* - (1 - \pi)r = \pi r\left\{1 - \frac{\gamma[1-\bar{G}][F-p+rk_o]}{rk_\mu}\right\},$$

where k_μ is the average capital used by all firms – that is, $k_\mu \equiv \bar{G}k_\ell + (1 - \bar{G})[(1 - \gamma)k_s + \gamma k_o]$. Setting $F = p + k_o$, so that the fine takes all of the firm's assets, (37) becomes

$$(38) \qquad r^* - (1 - \pi)r = \pi r\left\{1 - \frac{\gamma[1-\bar{G}][1+r]k_o}{rk_\mu}\right\}.$$

At $\bar{G} = 0$ and $\gamma = 1$, (38) becomes $r^* - (1 - \pi)r = -\pi$. It is then clear that $r^* - (1 - \pi)r < 0$ for positive $\bar{G}$ that are sufficiently close to zero. If β is not too high, we will then also have $\beta h' + r^* - (1 - \pi)r < 0$ for $\bar{G}$ sufficiently close to zero. If we then lower γ below one, we can achieve

28

equality between $\beta h' + r^*$ and $(1 - \pi)r$ for $\bar{G}$ sufficiently close to zero (since lowering γ causes r to fall and if we take γ to zero, r would equal r^*). As $\bar{G}$ rises we can continue to find a γ that restores this equality (from eq. 19), until some maximum $\bar{G}$ is reached, at which point equality is obtained with $\gamma = 1$. It is then clear that $\bar{\alpha}_1(\beta)$ and $\bar{\alpha}_2(\beta)$ can be constructed with the properties in the proposition.   Q.E.D.

Thus, moving to a Becker equilibrium enables us to achieve greater compliance, with no change in inspection costs and no deadweight losses from inefficient use of capital, *if* the initial level of compliance is sufficiently low, and the capital-induced external cost (β) is not too high. Conditions for low compliance would include low external costs (both β and η), or high unit inspection costs.   But this result gives us a type of "Becker conundrum": the regulatory response to actions by firms that involve low social costs should be to save on inspection costs by bankrupting some firms and taking all of their assets.   But this result requires the qualification that β is sufficiently low. For high β, bankruptcy will be too costly from a social welfare viewpoint, because then the fine can no longer induce firms to use socially-efficient levels of capital.   The fundamental problem is that large fines are necessarily lump-sum punishments, if they cause bankruptcy.   In particular, the amount of punishment no longer depends on the amount of capital.

A possible set of efficient Becker equilibria is illustrated in Figure 3 by the curve called the "efficiency locus." At each point on this curve, the cost of capital is the same for overleveraged and solvent evaders.   Thus, they choose the same capital and therefore have the same costs when they are not inspected and fined.   For any partial Becker equilibrium, they must then be equally well off when detected and fined.   The solvent firms are able to pay investors back in full, leaving no assets for the firm owners, whereas the overleveraged firms pay a fine equal to the entire value of their assets (Prop. 1).   But the fine can no longer be uniquely determined by the level of capital, since all evaders have the same capital.   Thus, points on the efficiency locus are supported using a policy of

random fines, where the government chooses a fraction of firms γ to pay the high fine, with the remaining evaders paying the low fine. Given the marginal compliance cost $\bar{\alpha}$, this fraction is set so that r rises to the level at which all evaders pay the social marginal cost of capital.

The location of the efficiency locus can be used to show that points on the Becker frontier need not be full Becker equilibria. If β is positive, then the socially-optimal cost of capital for evaders should exceed the cost of capital for legal firms, since only the former create capital-related external costs. But then legal firms use more capital than evaders, so an increase in r makes evasion more attractive [see eq. 23 for the relation between r and $\bar{\alpha}$]. Thus, given π, the only way to get from the efficiency locus to the higher compliance level on the Becker frontier is for r to fall, and this is possible while maintaining equilibrium in the capital market (eq. 19) only if there are fewer overleveraged firms. It follows that points on the Becker frontier to the right of the efficiency locus are partial Becker equilibria, not full Becker equilibria. In other words, the level of compliance need not be maximized by going to a full Becker equilibrium.

For points on the Becker frontier where compliance is sufficiently high and the inspection rate π approaches one, it will not be possible to raise r enough for $r(1 - \pi)$ to equal or exceed r^*. In this case, legal firms are always more capital-intensive than overleveraged firms, so the Becker frontier is achieved with a full Becker equilibrium. We next ask whether such full Becker equilibrium is optimal.

8. When is a Full Becker Equilibrium Optimal?

In this section, we derive conditions under which a full Becker equilibrium is optimal. In particular, we shall consider cases where full Becker equilibria involve relatively high levels of compliance, in which case they lie on the Becker frontier. In such cases, it seems intuitively

reasonable for a full Becker equilibrium to be desirable when increasing compliance generates large social benefits. This turns out to be true, but with some qualifications that concern the cost side.

Any deadweight losses from moving to a full Becker equilibrium are given by $(1 - G(\bar{\alpha}))L$, where the deadweight loss L is defined in (25). Recall the quadratic approximation of this loss:

$$(39) \qquad L = \frac{1}{2}[\beta h' + r^* - (1 - \pi)r]^2 k',$$

where k' is the capital demand derivative. Since the "effective subsidy" on capital is squared in the deadweight loss formula, large values of β may imply a huge deadweight loss, relative to any gains from using fines that bankrupt firms to raise $\bar{\alpha}$ beyond its maximum Pigou value. We then have a tradeoff: going from the Pigou-optimal compliance level to a level in the Becker region reduces the number of firms that are creating external costs, but by increasing the capital used by the non-compliant firms, it increases the external cost per firm. If either β or the capital demand elasticity are low, then the latter consideration is unimportant, but a high capital demand elasticity may actually raise external costs so much that for high β, the Pigou optimum remains preferable to any Becker equilibrium. This demand elasticity is not an issue in Proposition 4, because although it increases the deadweight loss per firm, we also showed that it increases the rate at which firms become overleveraged as $\bar{\alpha}$ rises, given by $d\gamma/d\bar{\alpha}$, and the two effects cancel out. Since $\gamma = 1$ at a full Becker equilibrium, a higher capital elasticity increases the deadweight loss, without offsetting effects.

Consider, for example, the 2-technique case illustrate in Figure 4, where the technique with the low capital intensity is assumed to be socially efficient. Suppose that $\eta = 0$ and $\beta > 0$. If the capital intensities for the two techniques are sufficiently far apart, while the lower cost of capital faced by non-compliant firms in the Becker region causes them to switch to the high capital-intensive technique, then external costs will actually be higher in the Becker region than in the Pigou region.

On the other hand, similar intensities combined with a sufficiently high β will ensure that a Becker equilibrium is superior to the Pigou optimum.

If external cost depend only on the component of external cost not dependent on the capital intensity ($\eta > 0$; $\beta = 0$), then increasing this component enables us to increase the marginal benefit of additional compliance, $MB_{\bar{\alpha}} = [h'(x_n)(\beta k_o + \eta) - \bar{\alpha}]\bar{g}$, while having no effect on deadweight loss, L. Thus, sufficiently high η and low β will ensure that a full Becker equilibrium is better than the Pigou optimum. We provide a more formal statement of this result for the case of an iso-elastic marginal damage function.

Proposition 6: *Assume that the external damage function h is isoelastic. Then there exists positive numbers η' and δ, such that if external cost parameter η is greater than η', while external cost parameter β satisfies $\beta\eta < \delta$, then the globally-optimal fine and inspection rate supports a full Becker equilibrium, where all evaders are overleveraged.*

Proof: The iso-elastic h function may be written as $h(x_n) = (x_n)^{\theta}$, $\theta > 1$; in which case $\beta h'(x_n) = \theta[\beta^{\theta} + (\eta\beta)^{\theta-1}]y_n{}^{\theta-1}$. Thus, the constraints on η and β in the proposition place an upper bounds on deadweight loss, L, under any given structure of fines and inspection costs. On the other hand $MB_{\bar{\alpha}}$ can be increased without bound by raising η, insuring that increasing $\bar{\alpha}$ into the Becker region will eventually increase social welfare. Q.E.D.

Finally, it is tempting to conjecture that a sufficiently low p_a insures that the Pigou optimum provides greater welfare than any Becker equilibrium, but this conjecture turns out to be wrong. The problem is that the $\bar{\alpha}$ at which $d\pi/d\bar{\alpha}$ goes to infinity does not depend on p_a, so the maximum $\bar{\alpha}$ that satisfies the Pigou constraint is independent of p_a. If external costs are sufficiently high, it may be desirable to achieve higher compliance levels, at any unit inspection cost, including zero.

9. Examples

While we believe that our model applies to a broad spectrum of regulated activities, ranging from nuisances, such as product safety and illegal parking, to environmental, safety and financial. Not all of these regulation settings accord perfectly with the framework described above. Minor deviations may emerge if, for example, the regulation does not restrict the level of capital that firms may use.[15] Nevertheless, our main results with respect to the desirability of Pigouvian versus Becker regulation will carry through provided that the key features of the model remain in place. The key attributes of the model that we need are: a regulation that specifies some production technology, the existence of alternative technologies, the rental of some portion of capital from investors, the inability of investors to discern the legal status of their borrowers, random inspection and monetary fines that potentially bankrupt evaders. In addition, violations of the regulation must generate external costs that may depend upon the level of output, the type of capital or the amount of capital. To provide some additional context for the analysis that follows, we close this section by providing four concrete examples and with a brief description of how they satisfy the criteria needed to generate our results.

Ex. 1: Illegal parking by delivery firms.

Consider an industry of restaurants that deliver meals to households. When parking the delivery vehicle, the firm faces a regulation about where it may park legally. Generally, the costs of compliance will vary with the availability of legal spaces, congestion, the character of the neighborhood, and the type of delivery car used. Any firm that evades the regulation risks a monetary fine, which, if high enough, could lead to bankruptcy and the seizure of assets by the parking authority. Investors considering loaning vehicles or lending money to firms in this industry

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[15] If legal firms are free to use any level of capital, then an additional inefficiency arises in the partial Becker and full Becker cases since the increase in r above r^* will cause legal firms to under-invest in capital. This additional distortion makes the welfare analysis slightly more complicated, by adding additional welfare losses that are tied to strict punishment, but it does not alter the basic message of this paper.

will demand a premium that compensates them for their expected losses. Firms that face bankruptcy will perceive a lower marginal cost of capital and presumably substitute cheap capital for other relatively expensive inputs. In this example, the external cost depends only upon the amount of illegal parking. In order to reduce it, the government may use higher fines, more likely detection and perhaps a tax on the activity itself. Since it is the production of output—meal delivery—not the capital used to produce it—trucks, ovens, etc.—that produces the external cost, our analysis suggest the optimality of high fines, rather than high detection rates, particularly if we view illegal parking as belonging to the category of relatively "minor nuisances" (Proposition 5). This solution is based solely on efficiency considerations and assumes risk-neutrality.

Ex. 2: Pollution regulation

Pollution regulations often require the installation of the "best available" pollution control equipment to mitigate the damage from emissions. A firm can evade the regulation by choosing a different technology. Depending upon the age of the plant, the complexity of the production process, the firm's experience and the skills of its workforce, the cost savings will vary (forgoing these savings is equivalent , of course, to bearing the costs of compliance). High fines may expose the firm to bankruptcy if detected with the non-compliant equipment, but, as we have seen, they lower its marginal cost of capital, inducing the firm to use more capital-intensive production techniques than those employed by legal competitors. This factor market distortion is a cost of the severe fine. In addition, the external cost depends on the amount of non-compliant capital. If this particular cost is high, then low fines and frequent inspections may be desirable, so that that the fine structure can be adjusted to provide evaders of the regulation with the proper incentives to limit their capital usage. On the other hand, if there is little substitutability in the use of capital, then such incentives are unimportant (the deadweight loss L is low in our model), and high external costs then suggest the

use of high fines, particularly if high inspection costs make it costly to control pollution with low fines.

Ex. 3: Licensing requirements

Many industries have licensing requirements. For example, hair salons must hire licensed stylists. The licensing requirement typically specifies some minimal level of training (that is, a minimum level of industry-specific human capital).[16] Measure the human capital on the vertical axis of Figure 1. Then a non-compliant firm will hire stylists with illegally low levels of training and substitute other forms of capital, such as furniture, design and equipment, for the regulated human capital. Facing potential bankruptcy, these firms borrowed funds at a rate of $(1 - \pi)r < r$ and operate at a point such as B in Figure 1. Lenders will not perceive the compliance or non-compliance of the individual firm but will be aware that losses exist and demand the appropriate payment. The external cost will not depend upon the capital, but rather upon only the level of output. Assuming this external cost is not too high, Proposition 5 suggests the use of high fines.

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[16] In Michigan barbers must complete a 2,000 hour course of study. (Barbering Law Book, Michigan Department of Labor and Economic Growth, BCS-LDL-PUB-001 (02/06).

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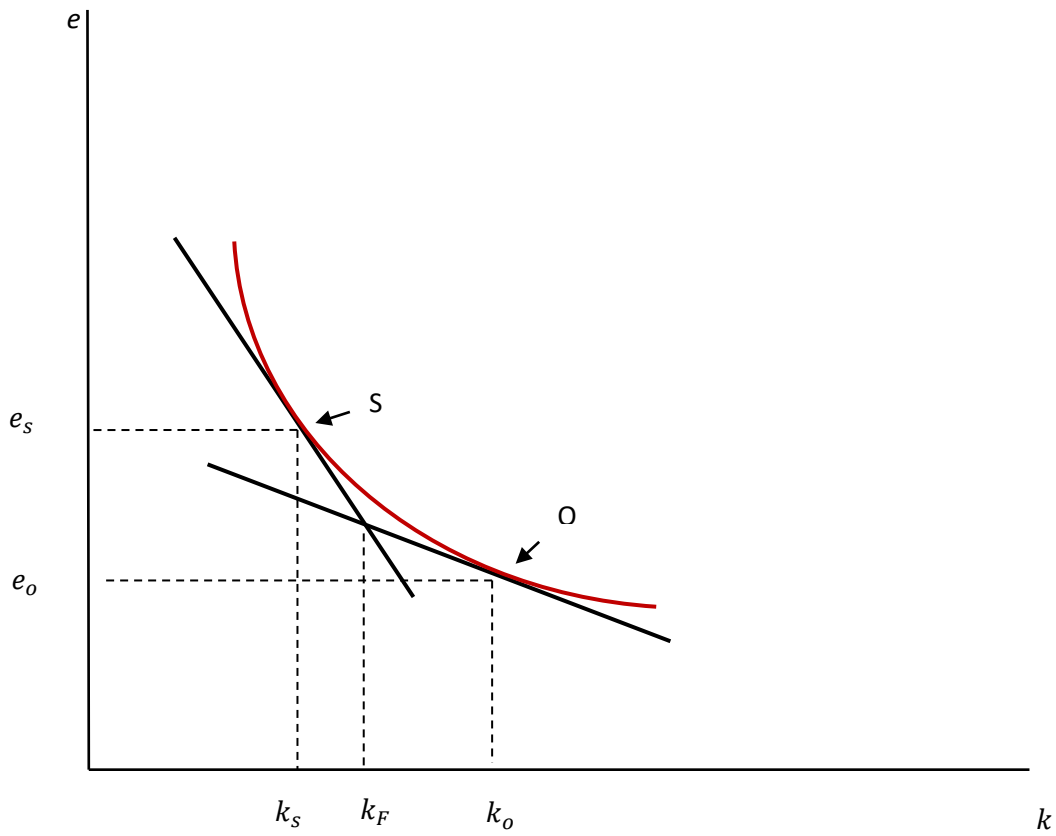
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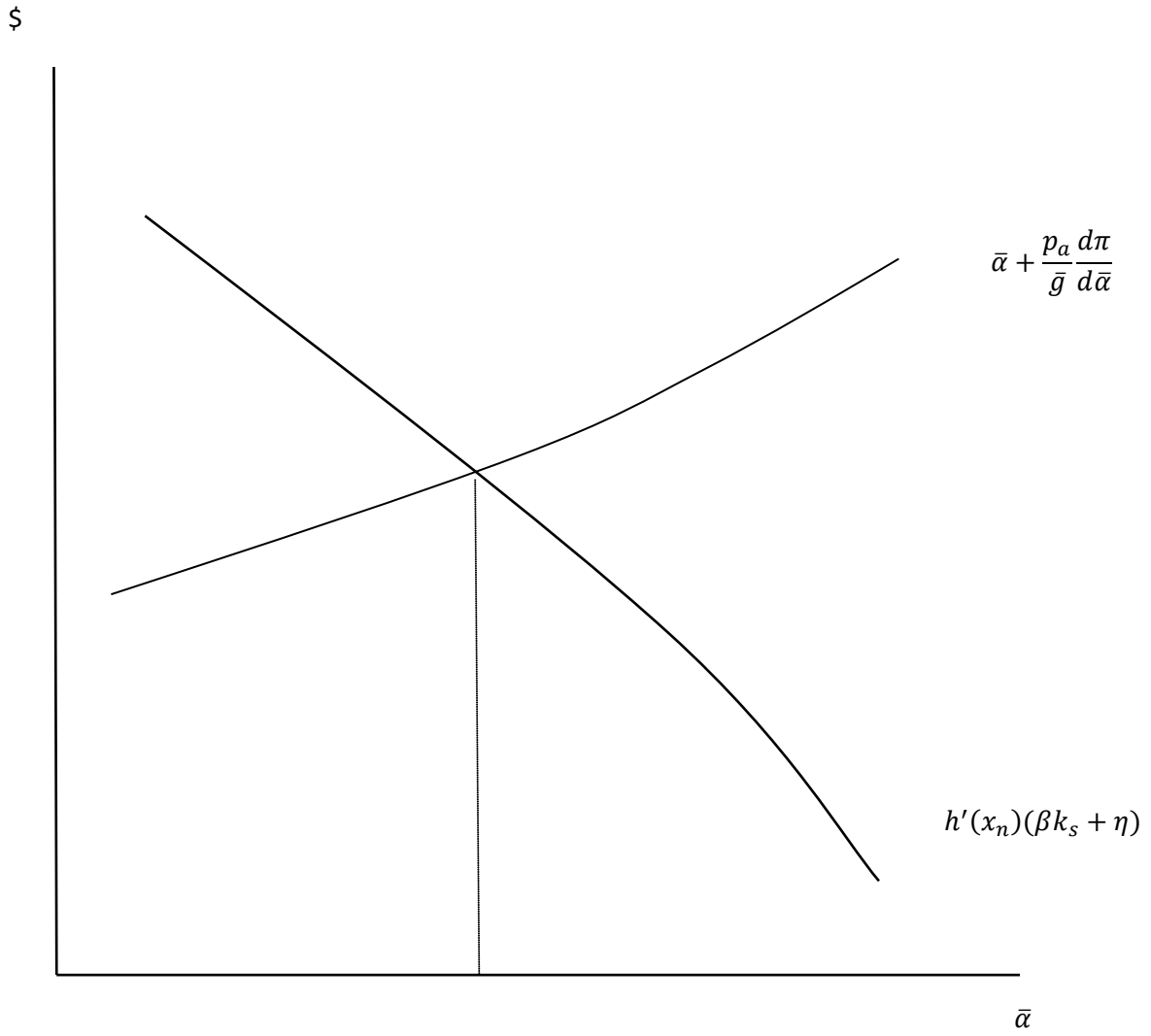
Figure 1: Choosing Inputs

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$$\bar{\alpha} + \frac{p_a}{\bar{g}}\frac{d\pi}{d\bar{\alpha}}$$

$$h'(x_n)(\beta k_s + \eta)$$

$$\bar{\alpha}$$
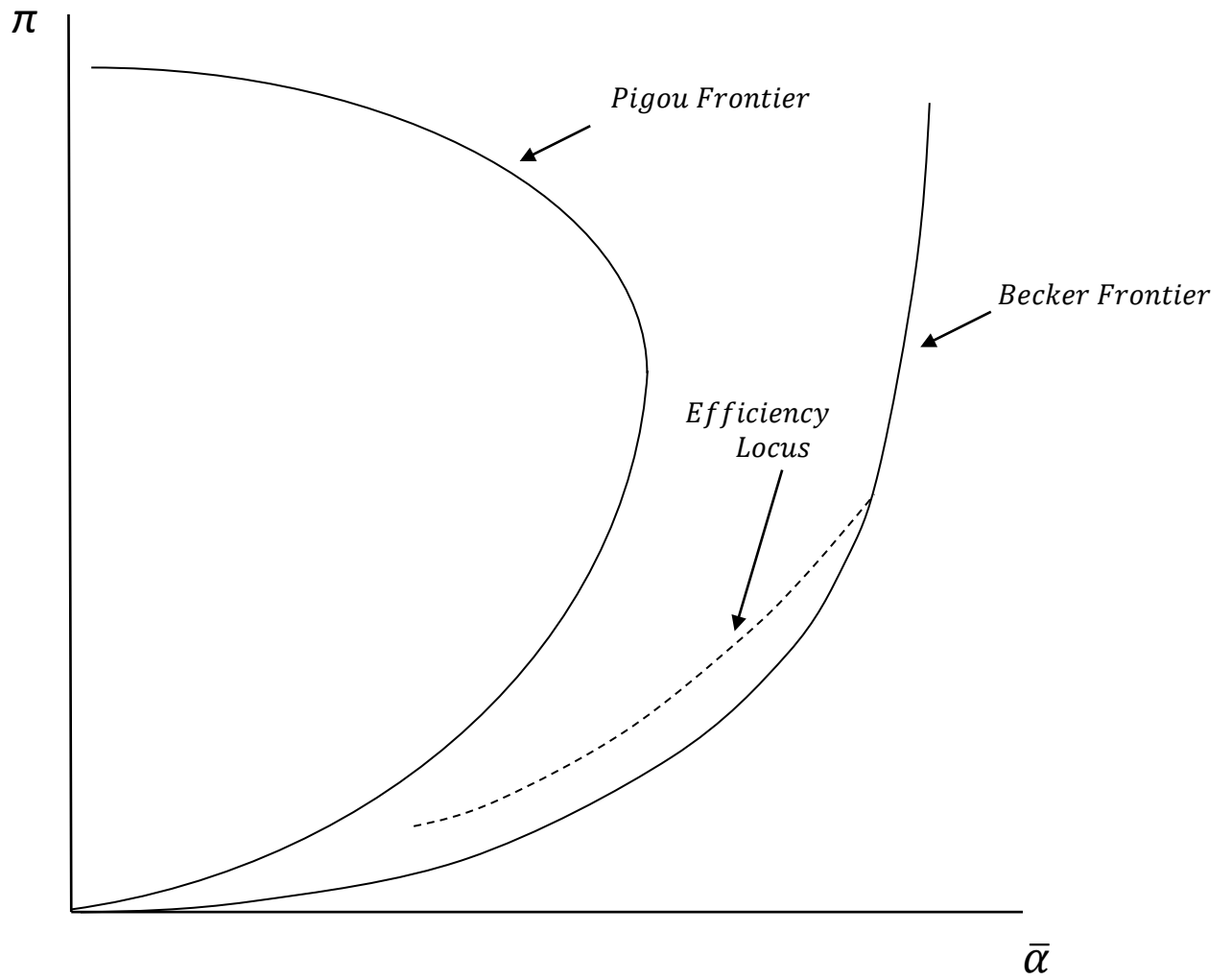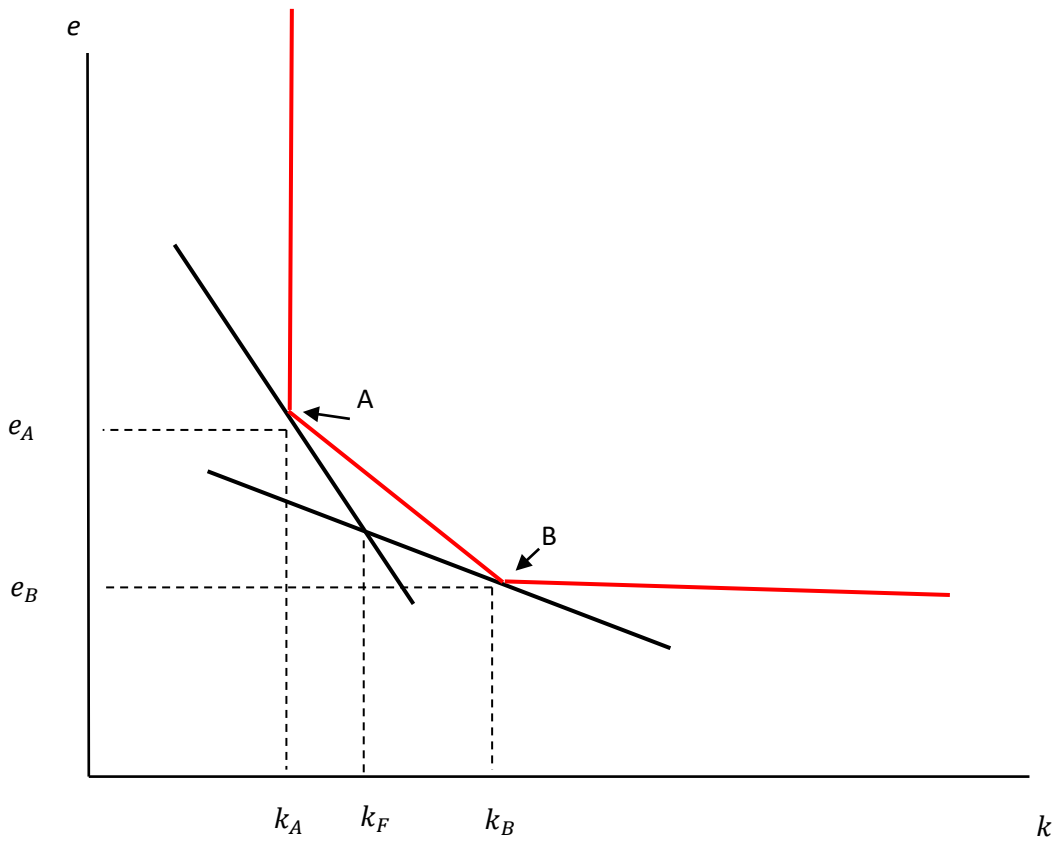
Figure 2: Optimal Pigouvian Regulation

Figure 3: Pigou and Becker Frontiers

Figure 4: Two Production Techniques