

Tax Evasion as an Optimal Tax Device

by

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January 2004

JEL Classification Code: H21, H26

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In his well-known textbook, Rosen (2005, p. 353) claims that a black market, or "underground economy," might improve welfare by effectively allowing some economic activities to be taxed at lower rate than others, in a manner consistent with optimal tax rules. We investigate this reasoning by introducing a black market into an economy where different goods are taxed at the same rate. We present conditions under which the black market moves the economy closer to an optimal discriminatory tax system, where goods are taxed at different rates. In some cases, the black market can be used to replicate this tax system.

Consider an economy with a continuum of consumers, indexed by a taste parameter, α . Each consumer is endowed with E units of a composite commodity, or "endowment good," which may be interpreted as labor. By supplying this endowment to competitive firms, a consumer obtains income that is used to purchase one or zero units of a variable-quality good, at a price equal to $P(\theta)$ for a quality- θ good, and $E - P(\theta)$ units of a consumption good. The utility function is

$$U(E - P(\theta), \theta; \alpha) = E - P(\theta) + \alpha v(\theta) \quad (1)$$

for a type- α consumer, where v is concave and public goods are suppressed because they are fixed in supply for the analysis. The parameter α possesses a continuous distribution, $h(\alpha)$, on $[0,1]$, and the population is normalized to equal one. From (1), higher values of α represent a greater marginal willingness to pay for quality. In this paper, we assume two qualities, θ_H and θ_L , with $\theta_H > \theta_L$, and we define $v_L = v(\theta_L)$ and $v_H = v(\theta_H)$.¹

The consumption good is produced from the endowment good, interpreted as "labor", by means of a constant-returns technology. In contrast, the variable-quality good is produced in fixed proportions from labor in capital, where a unit of capital is itself produced from one unit of labor. The critical difference between labor and capital is that capital is durable, remaining after production, whereas an hour of labor services spent in production is an hour unavailable for other uses. For simplicity, we assume no depreciation of capital, in which case all the capital is returned to consumers after production, at which point it is consumed. The consumer must be indifferent between supplying labor or capital. With the wage rate equal to one, the value of a unit of capital must equal one. Again using subscripts to denote goods, the quantities of labor, W , and capital, A , needed to produce a unit of each variable-quality good are W_H, W_L, A_H , and A_L .

¹ Davidson, Martin and Wilson (2004) conduct the lengthier analysis of the continuum case, which produces additional insights but not the conclusion that the black market replicates the optimal discriminatory tax system in some cases.

Before introducing tax evasion, we first derive the optimal discriminatory tax system, where low- and high-quality goods are taxed at separate rates to finance a given revenue requirement. Given the absence of depreciation and assuming a zero interest rate, zero profits in equilibrium implies that prices satisfy, $P_j(1-t_j) = W_j$ for $j = L$ and H , where t_j is the tax rate on good j . Let α_H be the α possessed by a consumer who is indifferent between the low- and high-quality goods, and let α_L be the α for a consumer who is indifferent between the low-quality good and consuming neither good. High- α consumers tend to choose a high quality. In particular, a consumer with an $\alpha > \alpha_H$ buys the high-quality good, a consumer with $\alpha \in [\alpha_L, \alpha_H]$ buys the low-quality good, and a consumer with $\alpha < \alpha_L$ buys neither good. Indifference between the low- and high-quality goods requires identical consumer surpluses: $\alpha_H v_H - P_H = \alpha_H v_L - P_L$, or,

$$\alpha_H = \frac{P_H - P_L}{v_H - v_L}. \quad (2)$$

For the consumer who is just indifferent between consuming the low-quality good and neither good, consumer surplus must be zero: $\alpha_L v_L - P_L = 0$, or

$$\alpha_L = \frac{P_L}{v_L}. \quad (3)$$

The optimal tax problem consists of choosing prices P_L and P_H to maximize total consumer surplus, subject to a government budget constraint:

$$\begin{aligned} \text{Max} \quad & \int_{\alpha_L}^{\alpha_H} (\alpha v_L - P_L) h(\alpha) d\alpha + \int_{\alpha_H}^1 (\alpha v_H - P_H) h(\alpha) d\alpha \\ \text{s.t } G = & (1 - \alpha_H)(P_H - W_H) + (\alpha_H - \alpha_L)(P_L - W_L), \end{aligned} \quad (4)$$

where G is the required revenue. The unit tax rates are then $t_H P_H = P_H - W_H$ and $t_L P_L = P_L - W_L$. From this problem, we obtain the following tax rule:

Proposition 1: *The optimal discriminatory tax system satisfies the following tax rule:*

$$\frac{t_H P_H - P_L}{P_H - P_L} = \frac{\Psi(\alpha_L)}{\Psi(\alpha_H)} \quad (5)$$

where $\Psi(\alpha) \equiv \frac{\alpha h(\alpha)}{1 - H(\alpha)}$. Furthermore, $t_L < (>) t_H$ if $\Psi(\alpha_L) > (<) \Psi(\alpha_H)$.

Proof: Attaching the Lagrange multiplier λ to constraint (4), we obtain the following first-order conditions for P_H and P_L :

$$\begin{aligned} \frac{\lambda - 1}{\lambda} &= \frac{h(\alpha_H)[t_H P_H - t_L P_L]}{[1 - H(\alpha_H)](v_H - v_L)} \\ &= \frac{h(\alpha_L)[t_L P_L - t_H P_H]}{[H(\alpha_H) - H(\alpha_L)](v_H - v_L)} + \frac{h(\alpha_L)t_L P_L}{[H(\alpha_H) - H(\alpha_L)]v_L} \end{aligned} \quad (6)$$

Multiplying (6) by a common denominator gives

$$v_L h(\alpha_H)[1 - H(\alpha_L)](t_H P_H - t_L P_L) - (v_H - v_L)h(\alpha_L)[1 - H(\alpha_H)]t_L P_L = 0. \quad (7)$$

If we multiply both sides of (7) by $(P_H - P_L)$ and make use of (1) and (2), then straightforward manipulation of (7) yields (5). #

Thus, which good has the higher optimal tax rate depends on the distribution of the taste parameter α . In particular, the government should tax the low-quality good more lightly if $h(\alpha_H)$ is small compared to $h(\alpha_L)$, or, in the case of similar densities, if the fraction of consumers buying legal goods, $[1 - H(\alpha_H)]/[1 - H(\alpha_L)]$, is large.

Suppose now that both goods are taxed at the same rate because it would be too costly for the government to collect the information required to implement the optimal discriminatory tax scheme. Firms may evade this tax by engaging in black-market activities, but they then risk detection and punishment.² Before capital suppliers are paid, the government audits a fraction π of firms and assesses monetary fines on an evading firm's assets. The firm's scale of production is irrelevant, given the assumption of a linear transformation between the endowment good and output. Thus, we may examine assets per unit of output sold. The value of these assets is the sum of revenue and capital, $P_i + A_i$. A fine is paid at the rate f on these assets. Since firms are risk neutral, competition drives their expected profits to zero: $P_i = \frac{W_i + \pi f A_i}{1 - \pi f}$.

Consistent with practice, we are assuming that the tax collector is first in line among the firm's creditors. Furthermore, the firm owners are unable to pass the burden of the fine on to labor by reneging on the payment of W_i and instead use this amount to pay the fine. Finally, fines higher than the firm's total assets are precluded by either the economy's legal system (e.g., limited liability) or the excessive costs needed to obtain them. To emphasize the desirability of a black market, even in the presence of low audit costs, we will allow audits to be costless.

² The analysis could be generalized by assuming that an exogenous percentage of income is not reported.

A firm that produces the high-quality good will choose to operate in the legal market only if the taxes associated with doing so are less than or equal to the expected fines associated with black market transactions; or,

$$\pi f(P_H + A_H) \geq tP_H. \quad (8)$$

Similarly, a firm producing the low-quality good will opt for the black market if

$$\pi f(P_L + A_L) \leq tP_L. \quad (9)$$

If these two inequalities are reversed, the black market is characterized by high-quality goods, whereas low-quality goods are offered in the legal market.

Using these inequalities, we now prove our main result.

Proposition 2: *If the low- (high-) quality good is labor intensive and $\Psi(\alpha_L) > (<) \Psi(\alpha_H)$ under a uniform tax system, then welfare can be increased by introducing a black market for the low- (high-) quality good.*

Proof: Consider the case where the low-quality good is labor intensive. (The other case is handled similarly.) Start with an optimal uniform tax system, where both goods are taxed at the rate t_H . We then introduce the black market for the low-quality good by setting πf so that the expected fine per unit of the low-quality good is slightly below the unit tax, that is, (9) holds strictly. The revenue loss is then offset by raising t_H slightly. For the high-quality good to also be produced in the black market, the inequality in (8) must be reversed. But in this case, the assumption on factor intensities, $\frac{A_H}{W_H} > \frac{A_L}{W_L}$, implies that $\frac{A_H}{P_H} > \frac{A_L}{P_L}$, so that (8) cannot be violated if the two sides of (9) differ only slightly. Hence, the high-quality good is produced in the legal market.

Under our assumptions, the initial uniform tax violates the optimal tax rule, with the right side of (5) exceeding the left side. This violation is easily seen to imply that a revenue-neutral tax change towards a lower t_L and higher t_H raises welfare. But we have shown that introducing the black market is equivalent to implementing a tax change of this type. #

Finally, we can show that the black market replicates the optimal discriminatory tax system in some cases. Without loss of generality, consider the case of a black market for the low-quality good. Suppose that the government sets the uniform tax rate equal to t_H (the tax rate that it would apply to the high-quality good under a discriminatory tax scheme), and πf at a rate that leads to the same effective unit tax rate as t_L (the tax rate that it would apply to the low-quality

goods under a discriminatory tax scheme).³ This system will replicate the optimal discriminatory tax system if low-quality firms choose to evade taxes and high-quality firms choose to operate in the legal sector. From (8) and (9) (using the fact that zero profits in the two markets imply that

$P_H(1-t) = W_H$ and $P_L = \frac{W_L + \pi f A_L}{1 - \pi f}$), this will be the case if

$$\frac{A_L}{W_L} \leq \frac{t - \pi f}{\pi f(1-t)} \leq \frac{A_H}{W_H} \quad (10)$$

Thus, the government is more likely to be able to use the black market to implement the optimal discriminatory tax scheme if the two technologies differ a great deal in terms of capital intensity or if the difference between t_H and t_L is not too large (otherwise all firms would opt for the black market) or too small (in which case all firms would be drawn to the legal market).

References

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- Rosen, Harvey S, 2005. *Public Finance*. New York: McGraw-Hill/Irwin.

³ To find the value for πf that is equivalent to t_L , we can set $\pi f(P_L + A_L)$ equal to $t_L P_L$ and solve for the expected fine. Doing so yields $\pi f = \frac{t_L W_L}{A_L + 2W_L}$.