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TRANSACTIONS COSTS, FRICTIONAL UNEMPLOYMENT AND TECHNICAL CHANGE IN THE MARKET TECHNOLOGY*

BY CARL DAVIDSON AND LAWRENCE MARTIN[1]

We consider a one-sector growth model in which factor-market frictions are described by a market technology linking the number of unemployed factors to the number of new jobs. We explore the consequences of technical change in this technology, focusing on the impact on efficiency and find that the relationship between the two depends on the labor intensity of the market technology. We also compare technical change in the market and production technologies and find that the relative importance of the two depends on the labor intensity of the market technology and the elasticity of factor supplies.

1. INTRODUCTION

It is widely accepted that market economies devote a substantial fraction of their resources to the problems created by costly exchange (Niehans 1987). In fact, Hirsleifer (1984, pp. 421–422) reports that in 1982 over 22 percent of the U.S. nonagricultural labor force was engaged in transaction-facilitating activities.[2] It is therefore not surprising that transactions costs, in one form or another, have been used to explain the existence of a large number of important institutional features of modern market economies. For example, this concept has been linked to the existence of firms (Coase 1937, Williamson 1981), the demand for money (Hicks 1935, Baumol 1952, Tobin 1956), the use of money as a medium of exchange (Karni 1973, Jones 1976, Niehans 1978), the failure of the market to reach an unconstrained Pareto efficient allocation (Coase 1960), the existence of equilibrium frictional unemployment (Diamond 1984, Howitt 1985, Friedman 1968) and the use of contracts to allocate labor services (Azariades 1975, Bailey 1974).

In many studies, transactions costs are modelled by making use of a "market" technology that describes the number of transactions that take place as a function of certain inputs. For example, when trading frictions are present in factor markets, the market technology describes the number of jobs created as a function of the search effort of unemployed factors. To date, virtually all studies of transactions costs focus on the implications of the assumption that this technology is not perfect (in that not all desired transactions take place). Yet, no attempt has been made to examine the impact of *changes* in the market technology. This is surprising for two

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[2] Using a more liberal definition of the "transactions sector," Wallis and North (1986) reach each more dramatic conclusions. They claim that by 1970, the transaction sector accounted for over one-half of U.S. GNP and over one-third of all U.S. employment!

reasons. First, there is the perceived importance of costly exchange as evidenced by the resources devoted to it in the economy and in the profession. Second, since it is well documented that the primary source of economic growth is technical progress, much work has focused on the implications of improvements in production technologies. Given the obvious similarities between production and market technologies, it is surprising that there has been no attempt to apply the insights obtained in the growth literature to this issue.

In this paper we explore the consequences of changes in a certain type of transactions cost. To do so, we develop a one-sector growth model with friction-ridden factor markets. The frictions provide the role for the market technology, and their existence also implies that in equlibrium some factors will be unemployed. Therefore, the framework that we develop also affords us the opportunity to examine the relationship between unemployment, welfare and capital accumulation. In particular, we can attempt to assess whether low-unemployment economies outperform their high-unemployment counterparts.

The paper divides into five additional sections. In Section 2, we develop the basic model which is in the spirit of the search models of Diamond (1982, 1984) and Mortensen (1982). As stressed by both authors, transactions costs in the form of trading frictions produce an environment in which external effects emerge and distort incentives. We therefore go on to investigate the efficiency of equilibrium in Section 3. In particular, we characterize the nature of the distortions and show that the labor intensity of the market technology determines the type of bias inherent in the market.

In Section 4 we turn to our main concern—the impact of changes in the market technology. We begin by comparing Hicks neutral technical change in the market and production technologies. We show that these two types of technical advance produce fundamentally different effects. In particular, while neutral technical change in the production technology has its full impact immediately and leaves relative factor shares unchanged, neutral technical change in the market technology takes time to filter through the economy and increases entrepreneurs' relative share. We then go on to consider the relative importance of technical change in the two technologies. This is accomplished by comparing two economies—one with a better market technology (so that there is less unemployment) and the other with a better production technology. We consider two cases. First, we require that the technologies differ in a manner that represents a neutral technical change with respect to the overall economy (this is made precise in Section 4). We then compare the levels of welfare attained in the two economies and show that the low-unemployment economy generates a more efficient outcome. In addition, we demonstrate that the labor intensity of the market technology determines whether the low or high-unemployment economy accumulates more capital. These results lay the groundwork for our second case—one in which the two economies have the same consumption possibilities. We show that the differences in welfare and capital accumulation achieved in these two economies closely mimic the differences uncovered in our first case.

In Section 5, we enrich our model by allowing for endogenous search effort and elastic factor supplies. We show that while most of our results generalize, our welfare conclusions need to be modified. In particular, we show that the wage elasticity of factor supply plays a key role in determining which economy operates more efficiently. Briefly, the low-unemployment economy is more likely to achieve a higher level of welfare if factor market supply is relatively inelastic. We summarize our results in Section 6.

2. THE MODEL

We begin with an outline of our model. We employ a one-sector growth model in which a single infinitely lived generation produces the consumption good using three factors of production: labor (L), entrepreneurship (E), and capital (K). Each firm is run by an entrepreneur who produces output by hiring one worker and the profit maximizing level of capital. The common production function is characterized by diminishing returns to capital. Due to factor-market frictions, it takes time for entrepreneurs with idle capital and unemployed workers to meet. A job consists of an entrepreneur supplying time and capital matched with a worker supplying labor. The market technology is represented by a function that relates the number of new jobs created to the number of idle workers and entrepreneurs. An improvement in this technology reduces the time spent searching by both labor and entrepreneurs.

Each agent inelastically supplies her endowment of "market time"(i.e., time spent working or seeking employment) to one of the factor markets. Income can be earned by searching *as* a worker (i.e., seeking to provide labor services) *or* by renting capital and searching *for* a worker to employ (i.e., providing entrepreneurial services). Entrepreneurs face a set-up cost in that they must rent a fixed amount of capital (K_s) to create a vacancy. Once the vacancy is filled, they may rent additional capital before producing. We use K_m to denote the level of capital rented by a matched entrepreneur. The putty capital is rented in a perfectly competitive, frictionless capital market, and entrepreneurs transform it into clay (i.e., a particular form). Since some capital must be rented before search, entrepreneurs differ from workers in that they bear a rental cost of capital during the search process. Moreover, entrepreneurs face additional risk in that at any time their capital faces a probability (δ) of becoming obsolete. If this occurs, they must rent another K_s units of capital, give it form, and reenter the search process. This assumption captures the notion that one of the roles of an entrepreneur is to bear the risk that the business venture might fail due to random shocks. Equilibrium is efficient in that the market produces the right mix of workers and entrepreneurs only if this risk bearing activity is rewarded adequately.

Once the unemployed factors meet, they negotiate over how to split the revenue generated by the sale of their output. Their income can be used to purchase consumer goods and/or contribute to savings. We assume that agents choose a consumption plan to maximize expected utility with savings leading to capital accumulation in the usual manner. Finally, we assume that there is no population growth and focus on steady-state equilibria so that the rate of accumulation is just sufficient to replace the capital that becomes obsolete.

A. Consumer Behavior. The model consists of a large number of identical, risk neutral consumers (N) who choose consumption, $c(t)$, to maximize expected lifetime utility subject to an intertemporal budget constraint and an initial wealth constraint. Formally, the consumer's problem is to choose $c(t)$ to maximize (1) subject to (2) given initial wealth (held in the form of capital), K_i^0

$$U_i = E_t \int_0^\infty u[c_i(t)] e^{-\rho t} dt, \tag{1}$$

$$\dot{K}_i(t) = [r(t) - \delta] K_i(t) + Y_i(t) - c_i(t). \tag{2}$$

We denote individual i's wealth at time t as $K_i(t)$ and the flow of income as $Y_i(t)$. The latter is a random variable that depends upon the individual's occupation and employment status. For example, $Y_i(t)$ is the wage for employed workers, profit for employed entrepreneurs, and zero for unemployed factors. The gross interest rate is $r(t)$. Thus, $r(t) - \delta$ is the net return to savings. Finally, ρ is the subjective discount rate.

Given risk neutrality, there is a simple solution to the problem above. If $r(t) - \delta > \rho$, the net return to savings dominates the increase in utility from extra consumption today and agents save all their income. If $r(t) - \delta < \rho$, agents maximize utility by saving nothing. Only if $r(t) - \delta = \rho$ will consumers be willing to save and consume in the same period.[3] Therefore, in equilibrium

$$r - \delta = \rho. \tag{3}$$

B. The Market Technology. Search is required to find employment or to fill a vacancy. Once an unemployed worker and an entrepreneur with idle capital meet, they remain matched until the capital becomes obsolete.[4] As long as they are matched, they produce a flow of output $f(K_m)$. We assume $f'(K_m) > 0$ and $f''(K_m) < 0$.

The number of new matches at time t, $M(t)$, is given by the market technology, where the inputs are the number of searching workers, $L_s(t)$, and entrepreneurs, $E_s(t)$,

$$M(t) = M[L_s(t), E_s(t)]. \tag{4}$$

We assume that this function is twice differentiable, increasing in both arguments, concave, and exhibits constant returns to scale.[5]

Combined with the assumptions that all unemployed workers are equally likely to find a job and that all searching entrepreneurs are equally likely to fill a vacancy, the

[3] Note that this implies that r is independent of t.

[4] A job could also break-up if one of the partners quits. However, in equilibrium no agent will ever choose to sever a productive partnership.

[5] See Pissarides (1986) and Blanchard and Diamond (1989, 1990) for empirical support of the CTRS assumption and Chirinko (1982) for empirical support on the concavity assumption. The assumption of CRTS ensures that equilibrium will be unique.

market technology defines the matching probabilities as

(5a) $$q_l = \frac{M(L_s, E_s)}{L_s} = \frac{M(\lambda, 1-\lambda)}{\lambda} = q_l(\lambda)$$

(5b) $$q_e = \frac{M(L_s, E_s)}{E_s} = \frac{M(\lambda, 1-\lambda)}{1-\lambda} = q_e(\lambda)$$

where $\lambda = L_s/(L_s + E_s)$ denotes the labor intensity of the searching population. These employment probabilities represent an alternative way to summarize the market technology. Our assumptions concerning M imply that q_l and q_e are twice differentiable functions of λ and that $q_l'(\lambda) < 0 < q_e'(\lambda)$.

C. Output and Employment. Since each match consists of one worker and one entrepreneur, the number of matched workers, L_m, equals the number of matched entrepreneurs, E_m, and both are equal to the total number of matches, X. Furthermore, since each matched pair produces $f(K_m)$ units of output, total output is equal to $Xf(K_m)$. If we let L and E denote the total number of workers (searching and employed) and entrepreneurs (idle and active), then this implies that at each instant[6]

(6a) $$N = L + E$$

(6b) $$L = L_s + X$$

(6c) $$E = E_s + X$$

These equations state that each agent must be either a worker or an entrepreneur and that each agent must either be employed or searching.

The matching technology described above tells us the number of *new* matches that are created each instant. To calculate the total number of matches, we must also determine the number of matches that break-up each instant. Flows from employment to unemployment result from random shocks that render capital obsolete.[7] When a shock occurs, the partnership dissolves, and each partner must reenter the search process to seek new employment. All capital (idle and active) faces the same instantaneous probability (δ) of obsolescence so that, at each instant, δX matches break-up. Combining this information with (4) to (6) allows us to describe the time path of total matches,

(7) $$\dot{X} = M(L - X, E - X) - \delta X.$$

Equation (7) is, in essence, a dynamic production function that relates the supplies of labor and entrepreneurs to the rate of change of employment.

[6] The time argument has been suppressed for convenience.

[7] Obsolete capital has no scrap value.

D. Choosing an Occupation.[8] Agents choose their occupation by comparing the expected lifetime income of a searching worker with that of an entrepreneur. If we let w denote the wage, then, from the fundamental equation of dynamic programming, we have

$$\rho V_{is} = q_i(\lambda)[V_{im} - V_{is}] \qquad \text{for } i = l, e \tag{8a}$$

$$\rho V_{lm} = w - \delta[V_{lm} - V_{ls}] \tag{8b}$$

$$\rho V_{em} = f(K_m) - w - r(K_m - K_s) - \delta[V_{em} - V_{es}], \tag{8c}$$

where V_{ls} (V_{lm}) represents the present value of expected *lifetime* earnings for searching (employed) workers in steady-state equilibrium, respectively.[9] For entrepreneurs, the terms V_{es} and V_{em} should be interpreted as earnings gross of the set-up fee. In each equation, the discount rate multiplied by expected lifetime income equals the flow of current income plus the probability of changing employment status multiplied by the capital gain from such a change.

As in most general equilibrium search models, we assume that the wage splits the surplus from employment evenly between the partners.[10] For a type i factor, the value of employment is $V_{im} - V_{is}$ so that the surplus is evenly divided when w is chosen to equate $V_{lm} - V_{ls}$ and $V_{em} - V_{es}$. Using (8) we obtain

$$w = \frac{(\rho + \delta + q_l)[f(K_m) - r(K_m - K_s)]}{2(\rho + \delta) + q_l + q_e}. \tag{9}$$

As (9) indicates, the wage increases with q_l. Intuitively, when workers have a relatively easier time finding a match, they are in a relatively stronger bargaining position and can secure a larger portion of the mutual gains. For later use, we substitute (9) into (8) to obtain

$$V_{is} = \frac{q_i[f(K_m) - r(K_m - K_s)]}{\rho[2(\rho + \delta) + q_l + q_e]}. \tag{10}$$

With compensation defined, we can now sum (2) over all agents to derive the economy's aggregate budget constraint. We obtain

$$\dot{K} = Xf(K_m) - c - \delta K. \tag{11}$$

The economy has X filled jobs, each of which produces a flow of $f(K_m)$ units of

[8] As far as we know, the only other models in which agents choose the side of the market on which to search are due to Baye and Cosimano (1990) and Yavas (1992).

[9] The equations in (8) hold in steady state. Out of steady state, there is an additional term added to the right-hand side of each equation that captures the growth rate of expected lifetime earnings (see, for example, Hosios 1990a).

[10] See, for example, Diamond (1982), Mortensen (1982), and Pissarides (1984).

output. This output is either consumed (c), used to replace obsolete capital (δK) or saved with savings leading to capital accumulation ($\dot{K}$).

E. Profit Maximization. It is important to note that K_m is treated as a parameter in (8) to (11). This value should be interpreted as the profit maximizing level of capital. Formally, after hiring a worker, matched entrepreneurs enter the capital market and choose K_m to maximize expected lifetime income. Applying Bellman's Principle of Optimality, we find that V_{em} is maximized when (see eq. (8c))

$$f'(K_m) = r. \tag{12}$$

We assume that, for relevant parameter values, the K_m that solves (12) is strictly greater than K_s. Thus, when an entrepreneur hires a worker, he expands his capital stock. This captures the notion that firms with vacancies demand less capital than otherwise identical firms with no vacancies.

Finally, the aggregate capital stock, K, is split between searching and matched entrepreneurs. That is

$$K = XK_m + E_s K_s. \tag{13}$$

F. Equilibrium. Since there is no population growth in this model, we consider only steady-state equilibria. Therefore, we set $\dot{K}$ and $\dot{X}$ equal to zero in the equations above. In addition, in equilibrium, agents sort themselves such that the net return to each type of search activity is the same. Since searching entrepreneurs must rent K_s units of capital, this requires that

$$\rho V_{es} - rK_s = \rho V_{ls}. \tag{14}$$

If (14) does not hold, then all searchers choose the same occupation and no new matches are created. The addition of this equilibrium condition completes the model. The two market clearing conditions are (3), which ensures that the goods market clears, and (14), which ensures that the factor markets clear. Our assumptions concerning the utility function and the two technologies guarantee that equilibrium is unique and saddle-point stable.

In the next section, we investigate the efficiency properties of the market allocation. We are particularly interested in whether the market provides the right composition of market participants. This is equivalent to asking whether the equilibrium value for λ is equal to the value that a social planner would choose. Before turning to this issue, we first demonstrate how the equilibrium value of λ is determined.

We begin by combining (14) with (3) to obtain

$$\rho(V_{es} - V_{ls}) = (\rho + \delta)K_s. \tag{15}$$

Given ρ and δ, (3) and (12) define unique values for r and K_m. Once these values are fixed, (10) indicates that V_{ls} depends only on λ. As λ increases, workers have a

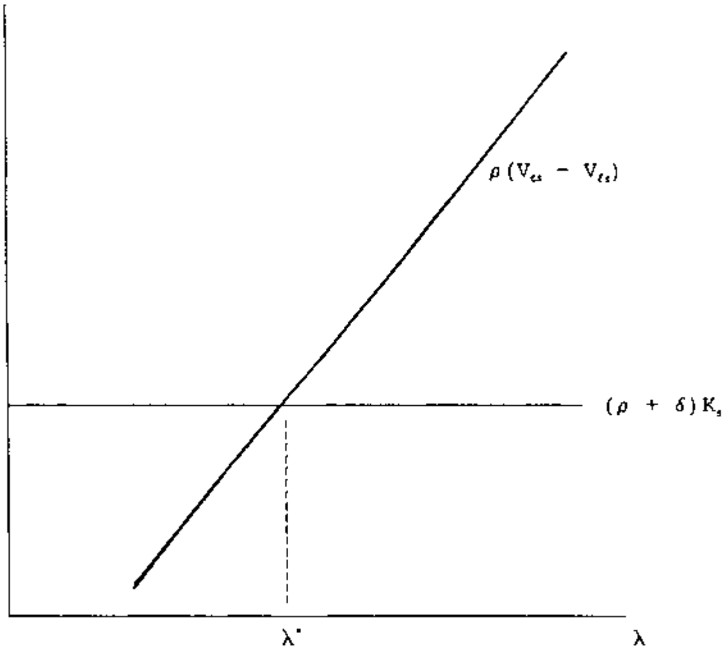

FIGURE 1

harder time finding employment while the prospects for searching entrepreneurs brighten. Therefore, the left-hand side of (15) is increasing in λ. This implies that for given values of ρ and δ there is a unique value of λ, defined by (15), that is consistent with steady-state equilibrium. This is illustrated in Figure 1.

We close this section with Lemma 1, which places a lower bound on the equilibrium value of λ. This result is crucial for the rest of the analysis.

LEMMA 1. *In equilibrium there are more workers than entrepreneurs. That is,* $L > E$. *Moreover,* $L_s > E_s$ *so that* $\lambda > 1/2$.

PROOF. Substituting (3) and (10) into (14) yields

$$\frac{(q_e - q_l)[f(K_m) - (\rho + \delta)(K_m - K_s)]}{2(\rho + \delta) + q_l + q_e} = (\rho + \delta)K_s > 0,$$

which implies that $q_e > q_l$. However, since $q_e = M/E_s$ and $q_l = M/L_s$, it follows that $L_s > E_s$. Finally, since $L = L_s + X$ and $E = E_s + X$, we have $L > E$. □

Intuitively, the market compensates entrepreneurs for bearing the cost of capital during search by offering them the brighter employment prospects that accompany minority status in the searching population. In equilibrium, the compensation is just adequate to ensure worker indifference with respect to occupation. We now turn to the question of whether the compensation is appropriate in terms of efficiency.

3. THE EFFICIENCY PROPERTIES OF EQUILIBRIUM

A. *Preliminaries.* Before tackling the optimal growth problem, we return to the dynamic production function introduced in (7) and use it to calculate the "marginal products" of labor and entrepreneurship. This can be done by first defining χ to be the normalized present value of expected output

$$\chi = \rho \int_0^\infty X(t) f(K_m(t)) e^{-\rho t}\, dt. \tag{16}$$

Note that in steady state, $X(t)f(K_m(t)) = Xf(K_m)$ for all t so that (16) yields $\chi = f(K_m)X$. Then, we can use the Diamond (1980) method to calculate the marginal products, where the term "marginal product" refers to the increase in the present value of output generated by a small increase in the factor supply taking into account the transition path between steady states. We obtain

$$\frac{\partial \chi}{\partial L} = \frac{M_l[f(K_m) - f'(K_m)(K_m - K_s)]}{\rho + \delta + M_l + M_e} \tag{17a}$$

$$\frac{\partial \chi}{\partial E} = \frac{M_e[f(K_m) - f'(K_m)(K_m - K_s)]}{\rho + \delta + M_l + M_e} - f'(K_m)K_s, \tag{17b}$$

where the subscript on M refers to the partial derivative of the market technology with respect to the factors. Note that there is an asymmetry in the marginal products. This reflects the fact that entrepreneurs pay the set-up cost required to create a vacancy. Applying (3) and (12), in equilibrium this set-up cost is composed of foregone consumption (captured by ρK_s) and depreciation (captured by δK_s). These marginal products are used below to characterize the set of efficient allocations.

B. *Optimal Growth.* The set of efficient allocations can be derived by summing (1) across consumers and then maximizing over $E(t)$ and $c(t)$ subject to the matching technology (eq. (4)), the aggregate budget constraint (eq. (11)), and the initial capital stock. Solving this growth problem yields the following necessary conditions for efficiency (these conditions are evaluated at a steady state)[11]

$$(\rho + \delta)K_s = \frac{(M_e - M_l)[f(K_m) - (\rho + \delta)(K_m - K_s)]}{\rho + \delta + M_l + M_e} \tag{18a}$$

$$f'(K_m) = \rho + \delta. \tag{18b}$$

[11] The optimal growth problem is described in detail in the Appendix of our working paper—Davidson and Martin (1992).

Equation (18b) determines the optimal K_m. Substituting this value into (18a) gives the efficient mix of workers and entrepreneurs (λ). Applying (3) and (12), this condition states that the marginal products of labor and entrepreneurship must be equal.

C. *The Market Allocation and Efficiency.* Comparing (18) with the equations defining equilibrium (eqs. (3), (12) and (15)) we find that equilibrium is efficient if and only if searching factors receive their marginal products. In general, this will not be the case since individual search decisions generate external effects. For example, an agent considering searching as a worker compares the present expected value of earnings with those of a searching entrepreneur, ignoring the effects of his decision on the employment prospects of other unemployed workers (q_l is decreasing in λ) and searching entrepreneurs (q_e is increasing in λ). These *congestion externalities* distort incentives and lead to sub-optimal behavior. For large N, even though each agent's decision has a very small effect on the welfare of others, these externalities do not disappear since the number of agents so affected is large. This point has been stressed in several articles on search and market performance (e.g., Diamond 1982 and Mortenson 1982).

Since congestion externalities always benefit one side of the market at the expense of the other side, equilibrium may entail too many or too few entrepreneurs. In the subsequent analysis the factor intensity of the matching technology, which we define as $\Theta_M = (M_l L_s - M_e E_s)/M$ plays a key role. The following lemma allows us to characterize the situations in which these externalities are positive or negative in the aggregate.

LEMMA 2. *Suppose that another worker is added to the economy. This entry generates positive (negative) congestion externalities in the aggregate if* $\Theta_M > (<) 0$. *The inequalities are reversed for searching entrepreneurs.*

PROOF. The addition of a worker has an aggregate impact on workers equal to $(\partial q_l/\partial L)L_s$. The aggregate impact on entrepreneurs is $(\partial q_e/\partial L)E_s$. Carrying out the differentiation and summing we obtain (see result A.1 of the Appendix) sign $[(\partial q_l/\partial L)L_s + (\partial q_e/\partial L)E_s] = \text{sign}[\Theta_M]$. □

Lemma 2 highlights the role of the factor intensity of the market technology in determining the market's bias. If, in equilibrium, the market technology is labor (entrepreneur) intensive, turning an entrepreneur into a worker benefits entrepreneurs more (less) than it harms labor. We would therefore expect the market to lead to a population that is not sufficiently labor (entrepreneur) intensive in this case.[12]

[12] The role of Θ_M in determining the efficiency of search equilibria was first uncovered by Hosios (1990b). However, he approached the problem from a different angle by investigating the conditions under which the division of the surplus created by a job leads to an efficient outcome (for a given market technology). He found that efficiency is achieved if labor's share is equal to $M_l L/M$. We take the division of the surplus as given (the partners split the surplus evenly) and ask when this leads to an efficient allocation. This occurs when $M_l L = M_e E$, which implies that $\Theta_M = 0$. Clearly, Hosios' analysis also applies to our model.

This conjecture can be confirmed by comparing the equilibrium conditions with those needed for efficiency. Equation (18a) indicates that λ should be set such that the marginal products of entrepreneurship and labor are equal. Equations (3) and (15) indicate that equilibrium occurs when the return to searching as an entrepreneur exceeds the return to searching as a worker by rK_s. Therefore, the equilibrium value of λ is efficient if the difference between marginal products is equal to the difference between the rates of compensation less the set-up cost of a vacancy. Applying (3), (10), (12), (14), and (17) to (18a) yields (see result A.2 of the Appendix)

$$\text{(19)} \qquad \frac{\partial \chi}{\partial E} - \frac{\partial \chi}{\partial L} - [\rho V_{es} - \rho V_{ls}] - rK_s = -\frac{(q_e + q_l)(\rho + \delta)}{\Delta}[f(K_m) - r(K_m - K_s)]\Theta_M,$$

where $\Delta = [\rho + \delta + M_l + M_e][2(\rho + \delta) + q_l + q_e]$.

As conjectured, (19) allows us to conclude that if $\Theta_M > 0$ in equilibrium, welfare could be enhanced by increasing the labor intensity of the work force above the level attained by the market. If, on the other hand, $\Theta_M < 0$ in equilibrium, welfare could be increased by reducing the labor intensity of the searching population. This result is local (in that Θ_M must be evaluated at its equilibrium value), and our analysis does not yet indicate which of these situations is likely to occur. We now turn to this issue.

D. *Unemployment and the Value of a Job.* In this subsection, we show that by making an assumption concerning the relationship between the unemployment rate and the value of a job, we can completely characterize the nature of the market's bias. This is accomplished by showing that this assumption allows us to sign the equilibrium value of Θ_M.

We begin by calculating the unemployment rate. In a steady state, the number of unemployed agents remains constant and is equal to the number of searching workers (L_s) plus the number of searching entrepreneurs (E_s). Since the number of market participants, employed *and* unemployed, is $2X + L_s + E_s$, the rate of unemployment is given by $\mu = (L_s + E_s)/(2X + L_s + E_s)$. In a steady state, $M(L_s, E_s) = \delta X$. Substituting for X and using the fact that the market technology exhibits constant returns to scale allows us to express the steady state unemployment rate as a function of λ

$$\text{(20)} \qquad \mu(\lambda) = \delta/[2M(\lambda, 1 - \lambda) + \delta].$$

Turn next to the value of a job. Each time an entrepreneur and a worker form a partnership, their expected lifetime incomes increase from V_{es} and V_{ls} to V_{em} and V_{lm}, respectively. Therefore, the total value of the match is given by $Z = V_{em} - V_{es} + V_{lm} - V_{ls}$. Using (8) we can express Z as a function λ

$$\text{(21)} \qquad Z = \frac{2[f(K_m) - (\rho + \delta)(K_m - K_s)]}{2(\rho + \delta) + q_l(\lambda) + q_e(\lambda)}.$$

A job is valued by individuals because employed factors can expect to earn more than searchers. In the presence of a high unemployment rate, it seems natural to expect the gap between what matched and unmatched factors expect to earn to be relatively large. We therefore assume that the total value of a job increases with μ. Differentiating (20) and (21) we obtain

$$\frac{\partial Z}{\partial \mu}=\frac{\delta Z^2[q_l'(\lambda)+q_e'(\lambda)]}{2\mu^2(M_l-M_e)}\left[f(K_m)-(\rho+\delta)(K_m-K_s)\right]. \tag{22}$$

It can be shown that this value is positive if and only if $\Theta_M^2 > M_l M_e(1-2\lambda)^2$ (result A.3 of the Appendix) so that this assumption bounds Θ_M away from zero for all $\lambda \neq \frac{1}{2}$. This bound allows us to sign Θ_M and therefore provides us with enough information to characterize the market distortions.

LEMMA 3. *Suppose that the value of a job increases with the unemployment rate, then it follows that*

(1) *if* $\Theta_M > 0$ *when* $L_s = E_s$, *then* $\Theta_M > 0\ \forall L_s, E_s$;
(2) *if* $\Theta_M < 0$ *when* $L_s = E_s$, *then* $\Theta_M < 0\ \forall L_s, E_s$;
(3) *if* $\Theta_M = 0$ *when* $L_s = E_s$, *then* $\Theta_M > 0\ \forall L_s < E_s$ *and* $\Theta_M < 0\ \forall L_s > E_s$.

PROOF. By definition $\Theta_M = M_l L_s - M_e E_s$ and therefore Θ_M is continuous in L_s and E_s. Combining this fact with the bound on Θ_M implied by our assumption that $\partial Z/\partial\mu > 0$ leads to (1) and (2). To prove (3), note that if $\Theta_M = 0$ at $L_s = E_s$ then it follows that $M_l = M_e$ at this point and $M_l < (>) M_e$ for all $L_s > (<) E_s$ (since M is concave). However, as shown in result A.3 of the Appendix, when $M_l > (<) M_e$ $\partial Z/\partial\mu > 0$ if and only if $M_l L_s^2 > (<) M_e E_s^2$. In each case, multiplying the right-hand side by M_l and the left-hand side by M_e preserves the inequality. Taking the square-root then yields the desired result. □

This lemma states that Θ_M can be signed *globally* by investigating the market technology at a single point. If, for example, $L_s = E_s$ implies $M_l = M_e$ (as would be true if M is symmetric), then, when the market is symmetric, workers and entrepreneurs are equally productive in search. In this case, Θ_M can be signed by comparing L_s and E_s. However, there is no reason to expect factors to be equally productive in search. In general, attributes of factors that affect their physical productivity may affect their productivity in search (e.g., a real estate office may be more effective at filling a vacancy than an unemployed real estate agent is at finding a job). Cases (1) and (2) handle these possibilities. If, in symmetric markets, workers (entrepreneurs) are more productive in search, then Θ_M is always greater (less) than zero. Thus, it is useful to think of Θ_M as a measure of the symmetry of the market technology when evaluated at $L_s = E_s$. Proposition 1 shows how this measure can be used to characterize the nature of the market's bias in allocating factors.

PROPOSITION 1. *If entrepreneurs are at least as effective in search as workers in symmetric markets (i.e.,* $\Theta_M \leq 0$ *when* $L_s = E_s$*),then equilibrium is characterized by a searching population that is too labor intensive. A small decrease in proportion of* *workers would increase welfare. If labor is more productive in search in symmetric markets (i.e.,* $\Theta_M > 0$ *when* $L_s = E_s$*), welfare would rise with an increase in the labor intensity of the work force.*

The proof follows from Lemma 2 and 3 and the intuition is provided in the last paragraph of the preceding subsection. We conclude that the market allocation involves too little search by the factor that is most productive in the search process. This is so because agents on that side of the market ignore the aggregate net benefit generated by their search activity.

4. TECHNICAL CHANGE

A. *Hicks Neutral Technical Change in the Two Technologies.* In our model there are two technologies. First, as agents search, the market technology determines the number of jobs created and therefore, the number of agents capable of producing output. An improvement in this technology, holding the composition of the searching population constant, results in more jobs and output. Second, the production technology determines the amount of output that each matched pair can produce. An improvement in this technology, ceteris paribus, also increases output but does so in a different manner—by increasing output per job without altering employment.

These two technologies are depicted in Figure 2 where we have drawn the $f(K_m)$ isoquant for each function. The contour for the production function has the Leontief shape, requiring one worker and the entrepreneur with K_m units of capital to produce $f(K_m)$ units of output—additional entrepreneurs or workers are of no use. The $f(K_m)$ isoquant for the market technology tells us the number of market participants (employed and searching) of each type required to generate $f(K_m)$ units of output in a steady state. In addition to the two matched factors, searchers are required to produce jobs to replace those that break-up. To achieve a steady state with $X = 1$, the number of searchers would have to satisfy $M(L_s, E_s) = \delta$. Solving this expression for E_s as a function of L_s and adding one to each value (to account for the matched factors) yields the outer curve in Figure 2. This contour inherits its curvature from the properties of the market technology, and it lies to the right of the production isoquant since search utilizes resources.

Technical change in the production technology shifts both contours since it affects the number of matched agents required to produce a unit of output. For example, an innovation that allows a matched pair to produce $\eta f(K_m)$ units of output, with $\eta > 1$, would shift both isoquants towards the origin. On the other hand, technical change in the market technology affects only the number of searchers required to generate a given number of new jobs. Therefore, such a change shifts only the outer contour.

The goal of this section is to examine the impact of technical progress in the production and search processes on the economy. We are interested in two issues. First, there is the question of whether Hicks neutral change in these static technologies are neutral with respect to the dynamic technology defined by (7) and (16). If

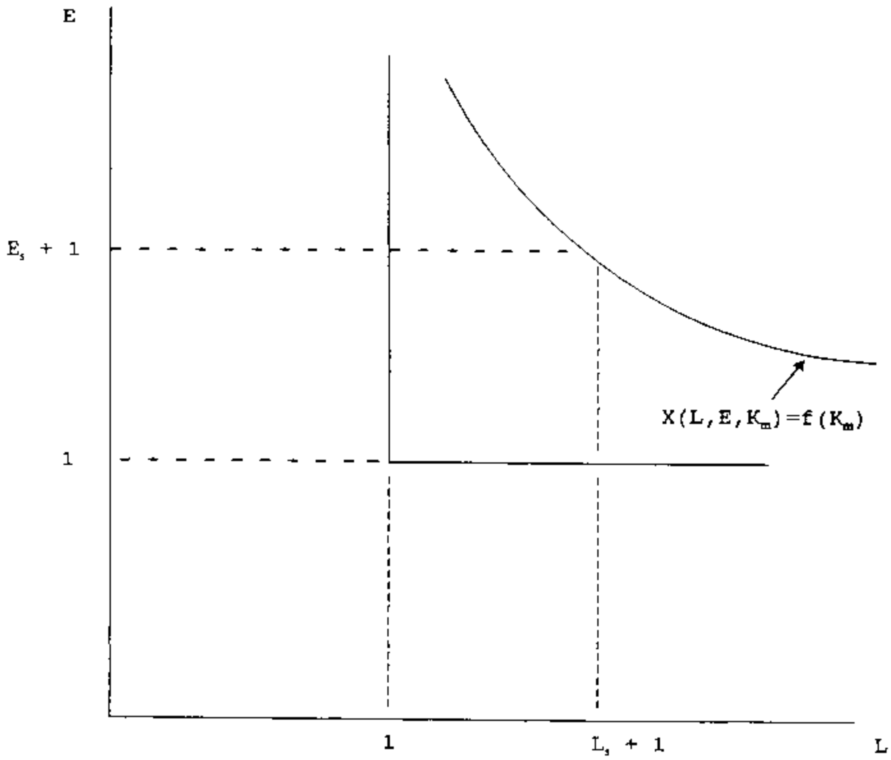

FIGURE 2

not, then the changes in the relative marginal products will have allocative effects. The second issue concerns the relative importance of the two types of technical progress. By this we mean the following. Suppose that we improve the market technology at the same time that we weaken the production technology and suppose that we do so in a manner that keeps the economy's steady-state consumption possibilities set constant (i.e., *potential* steady-state output is the same in both economies). How does this perturbation of the economy affect welfare and capital accumulation? Since unemployment is lower in the equilibrium generated by the perturbed economy, the answer to this question provides insight into the relationships between unemployment and capital accumulation and unemployment and welfare.

To examine these issues, we use η to denote the parameter measuring technical advance in the production technology and γ represents the degree of technical progress in the market technology. Therefore, a matched pair produces $\eta f(K_m)$ units of output each instant and the number of new jobs created is $\gamma M(L_s, E_s)$. By construction, an increase in $\eta(\gamma)$ produces Hicks neutral technological advance in the production (market) technology. In the initial steady state, $\eta = \gamma = 1$.

We begin by examining the impact of changes in η and/or γ on χ, the normalized value of steady-state output. In (16), $X(t)$ represents the number of jobs at time t so that total output is given by $\eta f(K_m)X(t)$. The time path of matches is still given by (7), although M must now be pre-multiplied by γ. Differentiating and evaluating at the initial steady state, we obtain

$$\frac{\partial \chi}{\partial \eta} = \frac{\partial \left[\rho \int_0^\infty \eta f(K_m) X(t) e^{-\rho t}\, dt \right]}{\partial \eta} = \chi = f(K_m) X. \tag{23}$$

Not surprisingly, a 1 percent increase in η increases output by 1 percent immediately.

On the other hand, improvements in the market technology disturb the steady state so that it is necessary to take into account the transition path between steady state in order to evaluate the full impact of such changes. Doing so and evaluating at the original steady state, we obtain

$$\frac{\partial \chi}{\partial \gamma} = \frac{M[f(K_m) - f'(K_m)(K_m - K_s)]}{\rho + \delta + M_l + M_e} \tag{24}$$

$$= \frac{\delta[f(K_m) - f'(K_m)(K_m - K_s)]}{\rho + \delta + M_l + M_e} X < f(K_m) X.$$

From (23) and (24) we conclude that improvements in the production and market technologies have fundamentally different impacts on the economy. While changes in $f(\ )$ have an immediate impact, it takes time before changes in the market technology produce an effect. This is due to the fact that search is required to produce jobs and, although improvements in the market technology can reduce time spent searching, it cannot eliminate it entirely.

Next, we turn to the question of whether these two types of Hicks neutral technical change are neutral with respect to the economy's dynamic technology, as defined by (16) and (7). Technical change is Hicks neutral if, for a given i/j with $i, j = L, E$ or K and $i \neq j$, it leaves the relative dynamic marginal products of i and j constant. If we calculate the dynamic marginal product of K and rederive the dynamic marginal products of L and E for given values of η and γ we obtain

$$\frac{\partial \chi}{\partial L} = \frac{\eta \gamma M_l [f(K_m) - f'(K_m)(K_m - K_s)]}{\rho + \delta + \gamma(M_l + M_e)} \tag{25a}$$

$$\frac{\partial \chi}{\partial E} = \frac{\eta \gamma M_e [f(K_m) - f'(K_m)(K_m - K_s)]}{\rho + \delta + \gamma(M_l + M_e)} - \eta f'(K_m) K_s \tag{25b}$$

$$\frac{\partial \chi}{\partial K} = \eta f'(K_m). \tag{25c}$$

We are now in position to prove the following proposition.

PROPOSITION 2. *Hicks neutral technical advance in the production technology is Hicks neutral with respect to the dynamic technology. Hicks neutral technical advance in the market technology is entrepreneur augmenting in that it increases the marginal product of entrepreneurship relative to that of labor and capital. Labor's marginal product also rises relative to capital.*

PROOF. From (25), increasing η increases the marginal products by a factor of η so that relative marginal products remain constant. Increasing γ has a more complex effect. First, it increases the marginal product of E more than it increases the marginal product of L. To see this, note that the symmetric terms in (25a) and (25b), which reflect the impact of a rise in L or E on the number of matches, both increase in equal proportions when γ rises. However, since the increase in γ does not affect the set-up cost of capital (which is reflected in the second term in (25b)) $\partial\chi/\partial E$ rises more than $\partial\chi/\partial L$. Second an increase in γ alters λ at the same time that it increases X. To see this, note that $L_s/E_s = (L-X)/(E-X)$ so that an increase in X with L and K held fixed alters the mix of the searching population. In fact, since $L_s > E_s$ in equilibrium (Lemma 1), increasing X increases L_s relative to E_s. This reduces M_l/M_e, further increasing E's relative marginal product. Both the marginal product of L and E increase relative to K since $\partial\chi/\partial K$ is independent of γ. □

Proposition 2 further underscores the fact that neutral technical change in the two technologies produces fundamentally different results. While neutral technical advance in the production function has an immediate impact and is neutral with respect to the economy's overall technology, the impact of technical advance in the market technology filters through the economy slowly and increases the marginal product of entrepreneurs relative to that of labor. The latter result is due (in part) to the fact that as more jobs are produced, searching workers and entrepreneurs are drawn out of unemployment in equal numbers. However, since $L > E$ in equilibrium (Lemma 1), this type of technical advance generates an even more labor intensive group of searchers. Of course, this reduces the relative marginal product of labor in search.

B. *On the Relative Importance of Technical Change in the Market Technology.* We are now in position to assess the relative importance of the two types of technical progress. Specifically, we are interested in whether economies with lower unemployment rates accumulate more capital or achieve a Pareto preferred time path of consumption (i.e., operate more efficiently) than their high-unemployment counterparts. In our framework, these issues can be addressed by comparing the steady-state equilibria reached by two different economies—one with a better market technology (and thus lower unemployment) and the other with a better production technology. This is accomplished by starting with an initial economy in a steady-state equilibrium and then perturbing the technologies by increasing γ and reducing η.[13] We then compare the equilibria achieved in the perturbed and

[13] Since we are comparing two economies, our derivatives should be interpreted as representing comparisons across steady states. If we were to compare two equilibria within an economy, we would need to take into account the transition path from one steady state to the other.

original economies. To make the comparison meaningful, we require that the changes in γ and η be chosen so that both economies possess the same steady-state consumption possibilities.

We proceed in two steps. First, we perturb the economy in a manner that keeps the dynamic marginal products of labor and entrepreneurial services constant. This allows us to determine how the perturbation affects the economy when the productivity of the economy's two service factors are held constant (in the absence of capital, this would constitute Hicks neutral technical change). We then move on to consider the perturbation that keeps potential steady-state output constant. By proceeding in this manner, we can separate out those effects that are due to changes in the marginal products from those that would arise even if the marginal products did not change.

For reasons that will become clear shortly, we consider a perturbation that keeps the dynamic marginal product of a worker and an *entrepreneur with capital* constant. Since an entrepreneur with capital has already paid the set-up cost of creating a vacancy, these marginal products are symmetric (see (25a) and (25b), where the marginal product of an entrepreneur with capital is given by the first term on the right-hand side of (25b)). This symmetry is important since it implies that if η and γ are changed in a manner that keeps one of the marginal products fixed, the other marginal product will remain fixed as well. From (25a) and (25b), these values remain unchanged if

$$\frac{\partial \eta}{\partial \gamma} = -\frac{\eta(\rho+\delta)}{\gamma[\rho+\delta+\gamma(M_l+M_e)]}. \tag{26}$$

In our model, entrepreneurs perform two functions—they rent capital and they provide entrepreneurial services. Changing η and γ in a manner consistent with (26) holds the productivity of the second function constant. Therefore, perturbing the economy as dictated by (26) keeps the dynamic marginal products of the two services provided in the economy fixed. For this reason, we refer to such technical change as *service neutral.*

To investigate the impact of such a perturbation, return to the equilibrium condition described in Section 2.F which determines the equilibrium value of λ (eq. (15)). Differentiating the right-hand side of (15), applying (26) and evaluating at $\gamma = \eta = 1$, we find that

$$\operatorname{sign}\left[\frac{\partial(V_{es}-V_{ls})}{\partial \gamma}\right] = \operatorname{sign}[(E_s-L_s)\Theta_M]$$

so that the shift in the demand for capital (in Figure 1) depends on the factor intensity of the market technology ($L_s > E_s$ by Lemma 1). If $\Theta_M > 0$ the upward sloping curve in Figure 1 shifts down and to the right resulting in an increase in λ. Proposition 1 indicates that this movement in λ enhances efficiency. Of course, if $\Theta_M < 0$ the curve shifts in the opposite direction and lowers λ. However, from Proposition 1 this once again results in λ moving in the direction that increases welfare. We can conclude that the perturbed economy generates a more efficient equilibrium mix of searchers than its high-unemployment counterpart. Our results are summarized in Propositions 3.

PROPOSITION 3. *Suppose that one economy, the "low-unemployment" economy, has a better market technology while its counterpart, the "high-unemployment" economy, has a better production technology.*[14] *Suppose also that the dynamic marginal product of a worker is the same in both economies, as is the dynamic marginal product of an entrepreneur with capital. Then the low-unemployment economy generates a more efficient factor mix. Moreover, if workers are at least as effective in search as entrepreneurs in symmetric markets (i.e.,* $\Theta_M \geq 0$ *when* $E_s = L_s$*), then capital accumulation is higher in the high-unemployment economy than it is in the low-unemployment economy. If workers are less effective, then capital accumulation could be higher in either economy.*

The proof of the efficiency result is outlined above. To compare capital accumulation we must compare λ and K_m in the two economies. In the $\Theta_M \leq 0$ case, we found that λ would be higher in the low-unemployment economy. This implies that there will be fewer entrepreneurs in the low-unemployment economy. Moreover, these entrepreneurs will use less capital per job since this economy also has a weaker production technology. As a result, the low-unemployment economy accumulates less capital than its high-unemployment counterpart. In the $\Theta_M \geq 0$ case, the impact of the perturbation on λ is reversed so that there are more entrepreneurs in the low-unemployment economy. However, since these entrepreneurs still use less capital per job, the overall impact on capital accumulation is ambiguous.

In moving from the original, high-unemployment economy to the perturbed, low-unemployment economy there are two competing forces. First, as the market technology improves, the return to search (V_{es} and V_{ls}) rises. However, since entrepreneurs are more likely to find employment (Lemma 1), they benefit more than unemployed workers. This increase in $V_{es} - V_{ls}$ lowers λ. Second, as the

[14] Even though there are fewer factor market frictions in the perturbed economy, this does not immediately imply that equilibrium unemployment will be lower in this economy. After all, the two economies do not produce work forces of the same composition. When we refer to the perturbed economy as the "low-unemployment" economy, we mean that for a fixed value λ this economy would be characterized by a lower unemployment rate than the original economy. Nonetheless, it is worth noting that this economy is also likely to generate a lower *equilibrium* unemployment rate than the unperturbed economy. To see this, note that the unemployment rate (given in eq. (20)) is minimized when $M_l = M_e$. From Lemma 1 we know that in equilibrium $L_s > E_s$. Therefore, if $\Theta_M \leq 0$ at $L_s = E_s$ it follows that in equilibrium $M_l < M_e$ and the change in λ definitely lowers unemployment (when we move to the perturbed economy). If, on the other hand, $\Theta_M > 0$ when $L_s = E_s$ there are two cases to consider. If $M_l > M_e$ then once again the difference in λ across the economies guarantees that the unemployment rate will be lower in the economy with fewer frictions. If $M_l < M_e$ in equilibrium then the difference in λ works in favor of higher unemployment in the perturbed economy while the reduction in market frictions works in favor of a lower unemployment rate. This is the only case in which the perturbed economy *might* generate a higher equilibrium rate of unemployment.

production technology weakens, the return to search falls (since matched pairs produce less output). By similar logic, this reduces $V_{es} - V_{ls}$, causing λ to rise. If γ and η are altered in a manner that is consistent with service neutral technical change, the labor intensity of the matching technology determines which effect dominates. With $\Theta_M > 0$ the impact on the production technology dominates and λ rises. The results are reversed if $\Theta_M < 0$.

We now turn to the comparison of two economies that have different production and market technologies but yet generate the same steady-state consumption possibilities. To do so, we must find the rates at which η and γ must be altered in order to keep $\eta Xf(K_m)$ constant. We begin by noting that in a steady state it must be the case that $\gamma M(L_s, E_s) - \delta X = 0$ or, alternatively, $\gamma M(N - E - X, E - X) - \delta X = 0$. Differentiating, we find that in a steady state

$$\frac{\partial X}{\partial \gamma} = \frac{\delta X}{M_l + M_e + \delta}. \tag{27}$$

Steady-state output is given by $\eta Xf(K_m)$. Differentiating this expression and making use of (27), we find that potential steady-state output remains constant if η and γ are changed such that

$$\frac{\partial \eta}{\partial \gamma} = -\frac{\delta\left[f(K_m) - f'(K_m)(K_m - K_s)\right]}{f(K_m)\left[M_l + M_e + \delta\right]}. \tag{28}$$

To investigate how such a perturbation affects welfare and capital accumulation, we return to the equilibrium condition, differentiate, and apply (28). From the worker indifference condition (eq. (15)) we obtain

$$\text{sign}\left[\frac{\partial(V_{es} - V_{ls})}{\partial \gamma}\right] = \text{sign}\left[2\rho L_s E_s(M_l + M_e) + \delta(E_s - L_s)\Theta_M\right].$$

From above, we know that the second term captures the impact of service neutral technical change. Therefore, the first term captures the impact of changes in factor productivity. Comparing (26) and (28), we find that, for a given change in γ, the reduction in η required to keep the marginal products constant is larger than the reduction in η required to keep steady-state output constant. Thus, when the economy is perturbed in a manner that keeps potential steady-state output constant, the marginal products of L and E rise. These higher marginal products result in increases in the expected return to search with V_{es} rising faster than V_{ls}. As above, this is due to the fact that searching entrepreneurs have an easier time finding a match than searching workers. Thus, entrepreneurs benefit more from the increase in labor's productivity than workers benefit from the rise in the productivity of entrepreneurship. For $\Theta_M \leq 0$, the change in the marginal products leads to forces that work in the same direction as those uncovered above. Therefore, λ is lower in the perturbed (low-unemployment) economy and this enhances efficiency. However, if $\Theta_M > 0$, the two forces work in opposite directions and the impact on λ cannot be signed. In the case of little discounting (ρ sufficiently low), the forces discussed above will dominate, λ will be higher in the perturbed (low-unemployment) economy and welfare will be enhanced.

This result is summarized in Proposition 4.

PROPOSITION 4. *Suppose that one economy, the "low-unemployment" economy, has a better market technology while its counterpart, the "high-unemployment" economy, possesses a better production technology. Suppose further that the two economies possess the same steady-state consumption possibilities. Then, if the discount rate is sufficiently low the low-unemployment economy generates a more efficient factor mix.*[15] *Moreover, if workers are at least as effective in the search process as entrepreneurs in symmetric markets, then the high-unemployment economy is characterized by more capital accumulation than the low-unemployment economy. If entrepreneurs are more effective, then capital accumulation could be higher in either economy.*

5. EXTENSIONS

In this section, we show how our analysis would be modified by the including elastic factor market participation and endogenous search effort. Since the results from the two modifications are similar, we concentrate on the former, limiting our discussion of the latter to a brief sketch.

A. *Endogenous Factor Market Participation.* We begin rewriting (1) assuming that the agent derives utility from consumption, suffers disutility from "market time" (i.e., time spent working or seeking employment) and that utility is separable in its two arguments,

$$U_i = E_t \int_0^\infty \{u[c_i(t)] + v_i[I_i(t)]\} e^{-\rho t}\, dt. \tag{29}$$

In (29), v_i, which varies across consumers, represents the disutility from market time with $I_i(t) = 1$ if agent i is employed or searching at time t and $I_i(t) = 0$ if agent i chooses to consume leisure instead. We assume that $v_i(1) < v_i(0)\ \forall i$. Factor market participation is then given by $N(t) = \sum_i I_i(t)$, which is now endogenous. Although $N(t)$ can only take on integer values, we assume that $N(t)$ is large for all t and treat it as a continuous variable.

In deciding whether or not to seek employment, agents compare the disutility from forgoing leisure, $v_i(0) - v_i(1)$, with the expected utility from seeking employment (ρV_{ls} or ρV_{es}). All agents who value leisure more than the return from search remain idle while all other agents enter one of the factor markets. Therefore, assuming that the agents are ordered by the marginal disutility from market time, the equilibrium value of $N(t)$ is defined by

$$v_i(0) - v_i(1) = \rho \max\{V_{ls}, V_{es} - (\rho + \delta) K_s\} = \rho V_{ls}, \tag{30}$$

[15] If $\Theta_M < 0$, this statement is true for all nonnegative ρ.

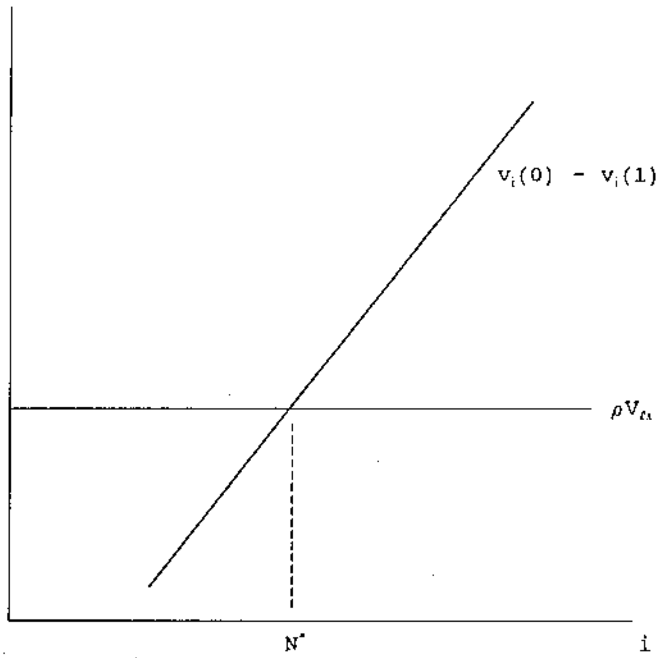

FIGURE 3

where i denotes the marginal agent (the second equality follows from the fact that, in equilibrium, agents sort themselves such that $V_{ls} = V_{es} - (\rho + \delta)K_s$).

This condition is illustrated in Figure 3. Since agents are ordered according to their disutility from time, $v_i(0) - v_i(1)$ is nondecreasing in i. In addition, although this curve is technically a step function, we have drawn it smooth due to our assumption that $N(t)$ can take on noninteger values. Note that since V_{ls} is independent of i, (30) defines a unique value for $N(t)$ for any given λ. Finally, it is straightforward to check that this extension does not alter any of the other equations of the model.

With $N(t)$ endogenous, efficiency requires that the dynamic marginal product of factor market participation be set equal to the disutility from foregoing leisure for the marginal agent. If we increase $N(t)$ by one unit, holding λ constant, the resulting dynamic marginal product is given by $\lambda(\partial\chi/\partial L) + (1-\lambda)(\partial\chi/\partial E)$. Thus, a social planner would set N such that

$$\lambda(\partial\chi/\partial L) + (1-\lambda)(\partial\chi/\partial E) = v_i(0) - v_i(1). \tag{31}$$

To determine if equilibrium is efficient, we compare the compensation offered by the market with the dynamic marginal product in (31). If we add λ workers and $(1-\lambda)$ entrepreneurs (so that N rises by one unit while λ remains fixed), the increase in compensation is given by $\lambda\rho V_{ls} + (1-\lambda)\rho V_{ls}$. Equilibrium factor market participation is efficient if this term is equal to the left-hand side of (31). Applying (10) and (17) yields

$$(32)\qquad \lambda\{\partial\chi/\partial L - \rho V_{ls}\} + (1-\lambda)\{\partial\chi/\partial E - \rho V_{ls}\}$$
$$= \lambda q_l(q_e - q_l)[f(K_m) - r(K_m - K_s)](\Theta_M/\Delta).$$

Since $q_e > q_l$ (Lemma 1), the sign of (32) depends on Θ_M. If $\Theta_M > 0$, adding λ workers and $(1-\lambda)$ entrepreneurs generates positive externalities in the aggregate, and there is too little factor market participation in equilibrium. In this case, the market offers searchers a level of compensation below their social value so that welfare would be enhanced by increasing N above its equilibrium level. The intuition follows from Lemmas 1 and 2—since there are more workers than entrepreneurs (Lemma 1), the externalities generated by adding entrepreneurs are always dominated by the externalities generated by adding workers (provided that searches are added in a manner that keeps λ fixed); and, from Lemma 2, adding a worker generates positive externalities in the aggregate if $\Theta_M > 0$. The argument is reversed if $\Theta_M < 0$.

While this extension does not affect Lemmas 1 to 3 or Proposition 2, it does require us to modify Propositions 1 and 3. With respect to Proposition 1, when entrepreneurs are at least as effective in the search process as workers in symmetric markets (i.e., $\Theta_M < 0$), the searching population is too labor intensive and too large. To increase welfare, N and λ must be reduced. In contrast, when $\Theta_M > 0$ welfare is enhanced by increasing N and λ.

When we compare low and high unemployment economies, we must now examine how the perturbation affects N as well as λ. For example, if we examine the impact of service neutral technical change (perturbing the economy in accordance with eq. (26)), we find that $V_{es} - V_{ls}$ and V_{ls} always move in opposite directions. This implies that if service neutral technical change increases λ, it decreases N (and vice versa). However, since N and λ need to move in the same direction to increase efficiency, the impact of service neutral technical change on welfare is ambiguous. We conclude that the first half of Proposition 3 generalizes only if the elasticity of factor supply is sufficiently low.

The fact that N and λ move in opposite directions implies the second half of Proposition 3 is unaffected by this extension. To see this, suppose that the perturbation increases λ and lower N (as would be the case if $\Theta_M > 0$). Since both effects reduce the number of entrepreneurs, the low-unemployment economy would be characterized by fewer entrepreneurs. Moreover, since this economy also possesses a weaker production technology, each entrepreneur uses less capital. As a result, the high-unemployment economy unambiguously accumulates more capital than its low-unemployment counterpart.

B. *Endogenous Search Effort.* The model can also be extended to allow for endogenous search effort without altering the qualitative nature of our results. We do so by assuming that the number of new matches created, M, depends on the total search effort of labor and entrepreneurs. If we let $z_{li}(z_{ei})$ denote the search effort of worker i (entrepreneur i) and use $\bar{z}_l(\bar{z}_e)$ to represent the mean search effort of workers (entrepreneurs), we can rewrite (4) and (5) as

$$(33)\qquad M = M(\bar{z}_l L_s, \bar{z}_e E_s)$$

$$(34a)\quad q_{li} = (z_{li}/\bar{z}_l)\left[M(\bar{z}_l L_s, \bar{z}_e E_s)/L_s\right] = z_{li} M(\lambda, 1-\lambda)/\lambda = q_l(\lambda) z_{li}$$

$$(34b)\quad q_{ei} = (z_{ei}/\bar{z}_e)\left[M(\bar{z}_l L_s, \bar{z}_e E_s)/E_s\right] = z_{ei} M(\lambda, 1-\lambda)/(1-\lambda) = q_e(\lambda) z_{ei}.$$

Note that the employment probabilities in (34) are increasing and concave in search effort. This extension also requires us to change the definitions of λ, the labor intensity of the searching population, and N, total factor market participation to $\lambda = \bar{z}_l L_s/(\bar{z}_l L_s + \bar{z}_e E_s)$ and $N = \bar{z}_l L_s + \bar{z}_e E_s$.

We use $c(z_{ji})$ to denote the search cost function (for $j = l, e$) and assume that is increasing and convex. In this framework, agent i chooses z_{ji} to maximize (the i subscript has been omitted for convenience since we only consider symmetric equilibria)

$$\rho V_{js} = z_j q_j(\lambda)(V_{jm} - V_{js}) - c(z_j) \qquad \text{for } j = l, e.$$

Therefore, applying Bellman's Principle, equilibrium search effort satisfies

$$(35)\qquad q_j(\lambda)(V_{jm} - V_{js}) = c'(z_j) \qquad \text{for } j = l, e.$$

That is, optimal search effort equates marginal cost and marginal benefit, where the latter is equal to the expected gain in lifetime consumption.

Resolving the planner's problem reveals that for efficiency search effort should be set such that its marginal cost equals its marginal product. Therefore, we can evaluate the efficiency of equilibrium as before—by comparing the dynamic marginal products of searching entrepreneurs and workers with the left-hand side of (35). Proceeding as above, we find that the difference between each factor's marginal product and the compensation it is offered by the market depends upon the sign of Θ_M. That is,

$$(36a)\qquad \text{sign}\{\partial\chi/\partial L - q_l(V_{lm} - V_{ls})\} = \text{sign}\,\Theta_M$$

$$(36b)\qquad \text{sign}\{\partial\chi/\partial E - q_e(V_{em} - V_{es})\} = \text{sign}(-\Theta_M).$$

The remaining analysis mimics that of Section 5.A with one qualification—all results must be stated in terms of total search effort rather than in terms of the numbers of workers and entrepreneurs. For example, Lemma 1 would state that in equilibrium total search effort by workers exceeds total search effort by entrepreneurs (i.e., $\bar{z}_l L_s > \bar{z}_e E_s$—so that it is still the case that $\lambda > \frac{1}{2}$). In Proposition 1, the term "symmetric markets" would now be interpreted to mean $\bar{z}_l L_s = \bar{z}_e E_s$

rather than $L_s = E_s$. As above, $\Theta_M > (<) 0$ in symmetric markets would still imply that both λ and N are too low (high).[16]

6. CONCLUSION

We have demonstrated that in the presence of factor market frictions, the elasticity of factor supply plays a key role in determining the relative desirability of improvements in the market technology. If factor supply is perfectly inelastic, then economies with fewer frictions operate more efficiently than comparable high-unemployment economies. However, if the elasticity of factor supply is relatively large, it is possible for high-unemployment economies to outperform their low-unemployment counterparts.

We have also shown that the equilibrium labor intensity of the market technology determines the market's bias and the nature of the relationship between unemployment and capital accumulation. Moreover, this factor is linked to the symmetry of the search process (Lemma 3). For example, if the marginal product in search is higher for labor than entrepreneurs when the market is symmetric, then the market technology is always too labor intensive. It therefore seems useful to consider which of the cases outlined above are more likely to represent reality and then summarize the results for this case.

From Lemma 3, in order to determine the equilibrium labor intensity of the market technology, we only need to consider the productivity of searching factors when the market is symmetric. We would argue that it is reasonable to assume that entrepreneurs are at least as productive as labor in the search process. After all, firms generally have access to a number of networks that are not equally accessible to labor that might reduce the amount of time it takes to fill a vacancy. In addition, if there is a learning curve associated with the search process, firms, which generally consist of a large number of job opportunities, are more likely to take advantage of the learning by doing possibilities. It is difficult to imagine any force that would provide labor with an advantage. Finally, in a recent paper Blanchard and Diamond (1990) attempted to characterize some of the properties of the aggregate matching function in the United States using data from 1968–1981. Their estimates reported in Table 1 of their article indicate a value for Θ_M below zero. Therefore, we consider $\Theta_M \leq 0$ to be the most likely outcome. In this case, Lemma 2 states that search by an additional worker would generate negative externalities while search by an additional entrepreneur would generate aggregate external benefits. This implies that the market allocation is characterized by a work force that is too large and labor intensive. Reducing market participation and increasing the capital intensity of the searching population would increase welfare. Moreover, in this case it is also

[16] As in the basic model presented in the text, our assumption of constant returns to scale in the market technology ensures that this model possesses a *unique* symmetric steady-state equilibrium in which agents choose to search. There is an additional equilibrium in which no one searches and all agents simply consume leisure. A proof of this proposition is available from the authors upon request.

likely to be true that a low-unemployment economy would accumulate more capital than its high-unemployment counterpart (provided the demand for capital is not too elastic).

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APPENDIX

RESULT A.1. Since $q_l = M_e(L_s, E_s)/L_s$ it follows that $\partial q_l/\partial L = [M_l L_s - M]/L_s^2$. However, since M exhibits constant returns to scale, $M_l L_s + M_e E_s = M$. Therefore, $(\partial q_l/\partial L)L_s = -(M_e E_s/L_s)$. Next, using the definition of q_e it follows that $q_e E_s = M$. Therefore, $(\partial q_e/\partial L)E_s = M_l$. Summing yields the desired result.

RESULT A.2. From (10) and (17a) we have (after applying eqs. (3) and (12)) $\partial\chi/\partial L - \rho V_{ls} = [M_l/(\rho + \delta + M_l + M_e) - q_l/(2(\rho + \delta) + q_l + q_e)][f(K_m) - r(K_m - K_s)]$. Let Δ denote the product of the two denominators. Then it follows that the right-hand side equals

$$(1/\Delta)\{(\rho+\delta)[2M_l - (M/L_s)] + q_e M_l - q_l M_e\}\{f(K_m) - r(K_m - K_s)\}.$$

Next, use the fact that $M = M_l L_s + M_e E_s$ and factor out $M(\rho + \delta)/L_s$ to write the first term in the brackets as $q_l(\rho + \delta)\Theta_M$. For the second term, apply the definition of q_i and then factor out $M^2/L_s E_s$ to obtain $q_e q_l \Theta_M$. Summing yields $(\Theta_M q_l/\Delta)[\rho + \delta + q_e][f(K_m) - r(K_m - K_s)]$.

Finally, the same argument can be used to show that $\partial\chi/\partial E - \rho V_{es} - rK_s = (\Theta_M q_e/\Delta)[\rho + \delta + q_l][f(K_m) - r(K_m - K_s)]$. Equation (29) then follows from subtracting these two expressions.

RESULT A.3. Differentiating q_l and q_e and using the fact that $M = M_l L_s + M_e E_s$ it is easy to show that $\text{sign}[q_l'(\lambda) + q_e'(\lambda)] = \text{sign}[\lambda^2 M_l - (1-\lambda)^2 M_e]$. Therefore, $\partial Z/\partial\mu > 0$ if and only if $(M_l - M_e)[\lambda^2 M_l - (1-\lambda)^2 M_e] > 0$. Carrying out the multiplication and factoring out Θ_M yields the desired result.

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