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Jobs and Chocolate: Samuelsonian Surpluses in Dynamic Models of Unemployment

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In dynamic models of unemployment in which the employed consume more than the unemployed, workers are finitely lived, and jobs are lasting, employment transfers consumption from future generations to those currently alive, resulting in a social surplus. That is, these transfers allow the current generation to consume more than its share of the output produced during its lifetime, without the increased consumption coming at the expense of future generations. Moreover, due to these intergenerational transfers, the allocation that maximizes steady-state output is Pareto dominated by another feasible allocation with a higher level of steady-state employment.

1. INTRODUCTION

Economics is rife with examples of important results that were derived in static models that may not hold in dynamic settings. One of the most celebrated examples of this phenomenon is Samuelson's (1958) result that in an overlapping-generations framework competitive equilibria are not generally Pareto efficient. In its simplest form, the logic is as follows.[1] Suppose that we have an overlapping-generations economy with zero population growth and no discounting in which agents live for two periods. In each period of life, agents receive an endowment of one unit of non-storable chocolate. For simplicity, assume that agents view consumption in the two periods as perfect substitutes. Then, it is easy to show that the competitive equilibrium entails each agent consuming his or her endowment in each period—that is, there is no trade. However, if, in every period, each of the young were to give one chocolate to each of the old, then everyone in the first generation would gain without anyone losing. This follows from the fact that the agents in every subsequent generation would receive a transfer when they are old that exactly offsets what they give up when they are young. Therefore, this transfer of chocolate across generations generates a Pareto improvement. Moreover, since the old agents consume more than the value of

1. See, for example, Shell (1971).

their endowment while the remaining agents consume output equal to their endowment, this policy creates a social surplus.[2]

In this paper, we argue that in dynamic models of unemployment in which the employed consume more than the unemployed, workers are finitely-lived, and jobs are lasting, such intergenerational transfers arise naturally and have important consequences that have heretofore gone unnoticed. In particular, we show that in the setting described, employment transfers consumption from future generations to those currently alive. This has at least two important implications. First, as a direct result of these transfers, a social surplus exists. As in the Samuelson example, the surplus implies that the current generation can consume more (in expected value terms) than the value of steady-state output, and that the increase does not come at the expense of future generations. However, in contrast to Samuelson's example, this surplus arises naturally in any steady-state and does not require outside (government) action to create it.

Second, the existence of these transfers implies that even when resources are allocated such that the value of steady-state output is maximized (call this allocation A), there exists another feasible allocation (A′) that Pareto-dominates it. This result arises from the fact that at A, the additional transfer created by a small increase in steady-state employment benefits members of the current generation without harming anyone. This occurs because at that level of employment, what each future generation gives to the cohort preceding it exactly equals what it receives from the cohort that follows it. Since the current generation receives a transfer from the unborn while giving up nothing in return, a marginal increase in employment results in a Pareto improvement.

To make these ideas more concrete, consider the following outline of a model of unemployment. In each period, there are L risk-neutral agents who are either employed or unemployed. These workers face a constant per period probability of death (d) so that life is finite and, upon death, each agent is replaced immediately by a newborn, unemployed worker.[3] Jobs last until either a worker dies or an exogenous shock causes a separation. Therefore, each period can be divided into three stages. First, the unemployed compete for jobs. Next, output is produced, traded, and consumed. Finally, some jobs break up. This last stage determines each worker's employment status for the beginning of the next period. We say that "jobs last" as long as some jobs survive across periods so that some workers begin the period with employment already secured.

For simplicity, assume that each job produces one unit of output, so that the number of jobs (X) is equal to total output. Then, in a steady state, if h represents the survival rate of a job, $(1-h)X$ new jobs must be created each period to replace those that dissolve at the end of the previous period.[4] Finally, let V_E represent the expected lifetime income

2. This scheme would not work if agents discounted the future since, in that case, agents would require a reward of more than one chocolate when old to offset the loss of one chocolate when young. However, as is well known, the social surplus can be generated in a model with discounting provided that the population grows at a rate that is no less than the discount rate (see, for example, Blanchard and Fischer (1989)).

More generally, when agents discount the future, the *timing* of consumption is just as important as the *amount* of consumption. Joel Fried (1980) illustrates this point in an overlapping-generations model where a technological improvement increases lifetime consumption but may reduce welfare for every generation after the first. The latter result follows since the exogenous shock is constructed in such a way as to shift income from youth to old age. If agents in his model neither discount the future nor experience diminishing marginal utility, they would delay all consumption to their old age. In this case, any innovation that expands output is necessarily welfare improving for all generations.

3. This corresponds to what Blanchard and Fisher (1989) refer to as a model of "perpetual youth."

4. The relationship between d and h will depend on the model. See the models in the text for specific examples.

for a worker who begins the period employed and let V_U denote the expected lifetime income for a worker who begins the period unemployed.

A key result that we demonstrate in the text is that, if all unemployed workers face the same probability of employment, then $V_U = \{X - c(X)\}/Ld$, where $c(X)$ denotes the social cost of producing X. That is, since $1/d$ is the expected lifetime of the representative worker, each *unemployed* worker can expect to earn an equal share of the net output produced during his expected lifetime. For the employed, however, expected lifetime income is given by $V_E = V_U + (V_E - V_U) = \{X - c(X)\}/Ld + (V_E - V_U)$. Each employed worker can expect to earn *more* than his share of the net output produced during the remainder of his (expected) lifetime.

Now, consider the aggregate income earned during the expected lifetime of the representative agent in this economy. The newborn cohort consists of Ld unemployed individuals, each of whom earn V_U. In addition, there are hX workers who begin the period employed. Each of these workers earns V_E. Finally, there are $\{(1-d)L - hX\}$ agents who were unemployed at the end of the previous period and survived to the current period. Since these individuals begin the period unemployed, they each earn V_U. Adding up the total expected lifetime income of all agents alive at the beginning of the period therefore yields

$$LdV_U + hXV_E + \{(1-d)L - hX\}V_U = \frac{X - c(X)}{d} + hX(V_E - V_U). \tag{1}$$

In the aggregate, expected lifetime income *exceeds* the value of expected net output, the difference being the social surplus.

To see how this surplus is created consider two steady-states characterized by different levels of employment, with $X_1 < X_2$. Although the mechanism at work is more complex, the transfer works roughly in the following manner. Since jobs last and since the employed consume more than the unemployed, a job today carries with it a claim on future output. An increase in steady-state employment therefore increases the current generation's share of future output. This comes at the expense of the newborns, who now earn a smaller share of output when they are young (they have a harder time finding jobs when they are young since they are crowded out of the labour market by the older workers). However, as the newborns mature and find employment they will eventually prefer the high-employment economy since they will then possess a greater claim on future output. Therefore, in moving from X_1 to X_2 the unborn sacrifice jobs when they are young in return for jobs when they are old. Whether they prefer the high- or low-employment economy depends on which effect dominates. Since the interests of the unborn are proportional to the value of steady-state output, these effects exactly offset each other at the employment level that maximizes $X - c(X)$.

The two implications follow immediately. First, the existence of intergenerational transfers creates the possibility of a Samuelsonian social surplus. In fact, as equation (1) indicates, a social surplus exists as long as there is positive employment ($X > 0$), jobs last ($h > 0$), and the employed consume more than the unemployed ($V_E > V_U$). Moreover, although it is not readily apparent from (1), the surplus would not appear if agents lived forever. After all, the surplus is created by borrowing from the infinite future and, if agents live forever, they would simply be borrowing from themselves (we prove this claim in Appendix A). Second, the transfer of consumption forward that is brought about by a marginal increase in employment above the level that maximizes $(X - c(X))$ benefits the current generation without harming the newborns (or future cohorts)—the social surplus is increased at no one's expense.[5] To compare our result to Samuelson's, it is as if the young are transferring jobs (rather than chocolates) to the old.[6]

We make these arguments precise in the remainder of the paper, proceeding in two steps. First, we introduce a simple model of a competitive economy with a fixed-coefficient technology. In this framework, equilibrium unemployment must be accompanied by a zero wage, and this implies that the employed and the unemployed earn the same income. Therefore, in equilibrium, no social surplus exists (in terms of equation (1), $V_E = V_U$). However, we show that any policy that transfers income from the unemployed to the employed results in a social surplus. In particular, we show that a minimum wage can generate a Pareto improvement. The mechanism at work is exactly the same as the one described above—the minimum wage transfers income across generations and allows the current generation to borrow from the future without harming anyone.

The advantage of the minimum wage model is its simplicity. Its disadvantages, for our purposes, are that no surplus exists in the laissez-faire equilibrium and that it is not possible to increase employment beyond its equilibrium level (so that it is impossible to show that a marginal increase would result in a Pareto improvement). In Sections 3 and 4 we turn to more complex models with a natural rate of unemployment. The unemployment in Section 3 is the result of trading frictions while in Section 4 it arises due to efficiency wage considerations. In both cases, we show that a social surplus exists for *any* positive level of steady-state employment. We also use the search model to show that the employment level that maximizes the value of net output is Pareto inefficient. Finally, we show that the surplus value from employment drives a wedge between the interests of the current and future generations. In particular, we show that the current generation is likely to prefer policies targeted at expanding employment above the level that would be optimal from the point of view of the unborn.

2. A MINIMUM WAGE ECONOMY

A. *Assumptions and equilibrium*

In order to illustrate formally the existence and nature of the social surplus attached to employment, we begin with a simple, discrete-time model of a competitive economy that produces one good (X) using capital and labour. The structure of the economy is as described in the introduction—each job lasts until either the worker dies or an exogenous shock causes a separation. If we use b to denote the exogenous separation rate, then $h = (1-d)(1-b)$.

5. This argument is informal and incomplete for the following reason. Let X^* denote the output level at which $X - c(X)$ is maximized. Then, provided that $X(V_E - V_U)$ is increasing in X at $X = X^*$, (1) indicates that the current generation receives a first-order gain when X is marginally increased above X^*. This change in output generates only second-order losses for the unborn, but there are an infinite number of unborn cohorts. Therefore, it is necessary to show that all losers can be compensated sufficiently when output is increased. We return to this issue in Section 3C.

6. Models without discounting and population growth provide the easiest framework in which to expose and explain the forces generating our results. We therefore restrict attention to such models in this paper. As in the Samuelson model (see footnote 2), all of our results generalize (and, in fact are strengthened) when agents discount the future provided that the population grows at a rate that is no less than the discount rate. If the population grows at a positive rate that is less than the discount rate, the flavour of our results generalize (a social surplus still exists in equilibrium), although there are some minor modifications that must be made (A may no longer be Pareto-dominated). Since additional issues arise when the growth rate exceeds the discount rate, we explore such models in greater detail in Davidson, Martin, and Matusz (1992).

We assume that each of the economy's L workers is endowed with α units of capital, so that the aggregate endowment of capital is $K=\alpha L$. Production is governed by a fixed-proportions production function of the form.

$$X=\min(K_E, L_E), \tag{2}$$

where K_E and L_E represent employed capital and labour. Cost minimization dictates that $K_E=L_E$. Since we are interested in economies characterized by unemployment, we assume that $\alpha<1$. Given this parameterization, capital will be fully employed but there will exist some unemployed labour in equilibrium. In particular, we have

$$K_E=K \quad \text{and} \quad L_E=\alpha L. \tag{3}$$

Once output is produced, it is traded in a perfectly competitive market. This implies that in equilibrium the value of output must equal the cost of production. Since each unit of output is produced using one unit of capital and labour, this condition is expressed as

$$1=r+w, \tag{4}$$

where w is the wage and r is the return to capital. The existence of unemployed labour forces the equilibrium wage to zero, thereby resulting in $r=1$.

Finally, consider the labour market. At the end of each period $L_E[d+(1-d)b]$ vacancies open up as some employed workers die and some employed workers who survive to the next period lose their jobs. Competing for these jobs are the $L_U(1-d)$ unemployed workers who survive, the $L_E(1-d)b$ employed workers who survive only to lose their jobs, and the $(L_E+L_U)d$ newborns. If we let π represent the steady-state probability of employment and assume that vacancies are filled from a random draw of the unemployed, then we have

$$\pi=\frac{L_E[d+(1-d)b]}{L_E[d+(1-d)b]+L_U}=\frac{\alpha(1-h)}{1-\alpha h}, \tag{5}$$

where the second equality follows from (3) and the fact that $L_U=L-L_E$.

B. *Expected lifetime income*

We measure expected income at the beginning of the period—before the vacancies are filled. If we use V_E and V_U to represent the expected lifetime income for employed and unemployed workers in a steady-state, respectively, then we have

$$V_E=w+\alpha r+(1-d)[(1-b)V_E+bV_U] \tag{6}$$

$$V_U=\pi V_E+(1-\pi)\{\alpha r+(1-d)V_U\}. \tag{7}$$

The derivations of (6) and (7) are straightforward. Consider the prospects of an employed worker. That worker earns $w+\alpha r$ with certainty this period. If he lives until next period and keeps his job, an event that occurs with probability $(1-d)(1-b)$, he continues to earn V_E. If he survives but loses his job, an event that occurs with probability $(1-d)b$, he earns V_U.

The expected income of an unemployed worker is derived in a similar fashion. At the beginning of the period, the worker has a probability π of finding a job, in which case he earns a stream of expected income equal to V_E. If he does not find a job, he earns the return on his capital this period, plus, if he survives to the next period (which occurs with probability $1-d$), he will begin unemployed and continue to earn V_U. Solving (6) and (7) for V_E and V_U and then using (5) to substitute for π yields.

$$V_E=\frac{\alpha r}{d}+\frac{w[\alpha b+d(1-\alpha b)]}{d(1-h)}=\frac{\alpha r}{d}+\alpha\frac{w}{d}+\frac{1-\alpha}{1-h}w, \tag{8}$$

$$V_U=\frac{\alpha r}{d}+\alpha\frac{w}{d}. \tag{9}$$

As already noted, in equilibrium $w=0$ and $r=1$. Therefore the expected lifetime income of an employed worker equals that of an unemployed worker, with the common value being α/d (the earnings from their capital endowment).

To find conditions under which a social surplus exists, we now sum the expected lifetime incomes of all workers currently alive. Of those alive at the beginning of the period, $(1-d)(1-b)L_E$ are employed. Each employed individual has an expected lifetime income of V_E. The remaining $L-(1-d)(1-b)L_E$ agents are unemployed, each earning an expected lifetime income of V_U. If V is the aggregate expected income of agents alive at the beginning of the period, then

$$V=(1-d)(1-b)L_EV_E+\{L-(1-d)(1-b)L_E\}V_U. \tag{10}$$

Substituting from (8) and (9), we can re-write (10) as

$$V=\frac{rK}{d}+\frac{wL_E}{d}+(1-d)(1-b)L_E\frac{w(1-\alpha)}{1-h}. \tag{11}$$

The first two terms on the right-hand side of (11) represent the total cost of production multiplied by the expected lifetime of the representative agent. Since the total cost of production must equal the total value of output, these two terms sum to X/d. The social surplus from employment is represented by the last term on the right-hand side of (11). This term can be rewritten so that

$$V=\frac{X}{d}+(1-d)(1-b)L_E(V_E-V_U). \tag{12}$$

Since $h=(1-d)(1-b)$ in this model, (12) is equivalent to (1) in the introduction. It is clear from this expression that as long as $V_E>V_U$ (i.e. the employed expect to earn more than the unemployed), a social surplus exists. However, in equilibrium, $V_E=V_U=\alpha/d$, so the last term on the right-hand side of (12) vanishes. Aggregate expected lifetime income is equivalent to the steady-state level of output multiplied by the expected lifetime of the representative agent. In the next sub-section we show that a minimum wage transfers income from the unemployed to the employed, driving a wedge between V_E and V_U and creating a social surplus. In addition, we show that a minimum wage is Pareto-improving.

C. *A Pareto-improving minimum wage*

What would be the effect of the government mandating a legally binding minimum wage? Consider any minimum wage $w\in(0, 1]$. From (4), the establishment of the minimum wage drives r to $1-w$. Since technology is characterized by fixed proportions, there is no change in K_E or L_E. By extension, there is no change in X. However, V now increases. Substituting these values into (11) (or (12)), we find that V is increasing in w

$$V(w)=\frac{X}{d}+hL_E\frac{(1-\alpha)w}{1-h}. \tag{13}$$

Furthermore, it is easy to show that the expected lifetime income of currently employed workers is strictly higher with the minimum wage when compared with the competitive equilibrium, while that of the currently unemployed is no different than under the competitive equilibrium. Substituting w and $r=1-w$ into (8) and (9), we have

$$V_E(w)=\frac{w(1-\alpha)}{1-h}+\frac{\alpha}{d}>\frac{\alpha}{d}=V_E(0), \tag{14}$$

$$V_U(w)=\frac{\alpha}{d}=V_U(0). \tag{15}$$

Since the employed gain without harming the unemployed, this policy unambiguously increases welfare.

Intuitively, the minimum wage harms the jobless as long as they remain unemployed, by reducing their return on capital. However, when they eventually do become employed, their wages will be higher. As (15) indicates, any minimum wage between 0 and 1 balances these two opposing forces. Moreover, from (14) the employed always gain from a higher minimum wage. Therefore, any $w<1$ is (weakly) Pareto-dominated by $w=1$.

Finally, it is possible to obtain a *strict* Pareto improvement relative to the competitive equilibrium by adopting a simple minimum wage/employment tax package. Suppose that the economy is in the laissez-faire equilibrium and that the following policy is proposed—a minimum wage is to be implemented and a small tax of T is to be imposed on anyone holding a job at the end of the first period. No taxes are to be paid by anyone in subsequent periods and the tax revenue is to be used to pay unemployment compensation to jobless agents in future periods. To be precise, measure time such that the proposal is made at the beginning of period $t=0$. Then, since the tax creates a pool of TL_E, an unemployed agent at time t can receive an unemployment benefit B_t, where[7]

$$B_t=\frac{sTL_E}{L_U}(1-s)^t \qquad s<1. \tag{16}$$

This policy costs each currently employed worker a small one-time payment of taxes that does not consume the entire expected benefit of the minimum wage and makes all unemployed workers strictly better off, since they now earn $B_t>0$ when unemployed and the minimum wage when they become employed. Therefore, the proposal would pass unanimously.

D. *Creating a social surplus*

To understand how the minimum wage creates a social surplus, we begin by rewriting V_U, the expected lifetime income of an unemployed worker, as

$$V_U=\frac{\alpha}{d}[r+w]=\frac{\alpha}{d}=\frac{\alpha X}{dK}=\frac{X}{dL}. \tag{17}$$

7. Since this policy does not affect employment, factor allocations, and so on, there is no transition period to consider. Note also that we must assume that X is storable for this proposal to work.

The first equality follows from (9), the second from (4), and the last two make use of the fact that $X = K = \alpha L$. Equation (17) indicates that the unemployed can expect to earn income equal to their share (i.e. $1/L$) of the output produced during their expected lifetime. Moreover, since X is independent of the wage, this remains true regardless of the value of the minimum wage.

Now, consider V_E, the expected lifetime income of an employed worker. We have

$$V_E = V_U + (V_E - V_U) = \frac{X}{Ld} + \frac{w(1-\alpha)}{1-h}. \tag{18}$$

Since $1 > \alpha$, the employed expect to earn more than their share of the output produced during their expected lifetime. Increasing the minimum wage allows the employed to increase their expected income without lowering the expected earnings of the unemployed. A *pure* social surplus is created.

It is obvious that the minimum wage transfers income from the unemployed to the employed. What is more subtle is that it also transfers income across generations. The reasoning is as follows. Because jobs are lasting, they carry with them a claim on future output. Moreover, since these jobs are held by members of the current generation while all newborns enter the labour market unemployed, the current generation will consume a disproportionate share of future output. As the wage increases, the current generation's share rises at the expense of the newborns, who now earn a smaller return on their capital—there is a transfer of income forward.

As long as the newborns remain unemployed, they are worse off. However, as they age and find employment, the minimum wage allows them to lay claim to a greater share of future output than they would earn in its absence. Therefore, the newborns sacrifice income when they are young (and unemployed) for extra income when they are old (and employed). The loss they suffer when they are young benefits the current generation while the subsequent gain comes at the expense of the cohort that follows them. Equation (17) indicates that the sizes of the intergenerational transfers are equal so that the newborns neither gain nor lose. In addition, since the future is infinite, this holds true for every subsequent generation as well. Just as in the Samuelson consumption-loan model, the current generation gains by borrowing from the infinite future.

There are three features of our model that are crucial to our argument: the overlapping-generations structure, the durability of a job, and the existence of two classes of agents *within* each generation (employed and unemployed) that consume different amounts of output. To see why, consider each feature separately, beginning with the overlapping-generations structure. In Appendix A, we extend our model to allow for infinitely-lived agents by introducing a discount rate (so that expected lifetime income does not explode). When we do so, (11) becomes

$$V = \frac{1+\rho}{\rho} X, \tag{19}$$

where ρ denotes the subjective discount rate. As (19) indicates, there is no surplus. Intuitively, since the surplus is created by borrowing from the future, without death, agents would be borrowing from themselves and could not gain.

Turn next to the durability of jobs. To demonstrate the importance of this feature, set the break-up rate (b) equal to one. As (12) clearly indicates, in this case, the minimum wage does not create a surplus. When a job lasts only one period, its holder is no more likely to be employed in the future than his unemployed counterpart. Consequently, employment today does *not* imply an increased claim on future output and a minimum wage does *not* transfer income across generations. With no transfer from the future, expected income cannot exceed the value of output and no surplus can be created.

The last feature concerns the distribution of output. With a minimum wage in place, the employed consume more than the unemployed. To see why this is important, consider what would happen if, after all vacancies are filled and production takes place, the government were to step in, collect all output, and distribute it evenly across all living agents. The aggregate expected lifetime income for the representative generation would then be

$$\frac{L}{d}\frac{X}{L}=\frac{X}{d}.$$

The surplus vanishes! Intuitively, by redistributing income the government is taking away the increased future consumption due to present employment from the current generation and giving it back to the unborn. This transfer from the current to the future obliterates the surplus. Finally, note that by abandoning this policy, the economy can achieve a Pareto improvement. This follows from the fact that those who are currently alive will consume more of the economy's output than those born in the future, while the next generation will receive its compensation as its members find lasting jobs and confer earnings in the more distant future. While the process is stochastic, it should be clear that this policy change increases the expected lifetime consumption of those currently employed without reducing the expected lifetime income of anyone else.

In summary, if the employed and the unemployed consumed the same amount of output, the current generation would not consume a disproportionate share of future output and there would be no transfer of income across generations (as in the laissez-faire equilibrium of this model). In the next two sections we consider models that allow for equilibrium unemployment in the presence of positive wage rates, and we show that no government action is required to create the social surplus.

3. A SIMPLE SEARCH MODEL

A. *Overview*

Our one-sector, discrete time search model is patterned after the continuous-time models of Mortensen (1982) and Diamond (1982).[8] While it shares the essential features of our minimum wage model, there are a few differences. In particular, unemployed workers must now expend effort searching to find a job and there is no capital.

Production of a unit of X now requires two agents. Accordingly, an unemployed individual must search for another unemployed worker in order to form a partnership. Once a partner is found, a match is created and, as long as it lasts, the partners sell the output they produce and split the proceeds evenly. The output is sold in a perfectly competitive market and we use this consumption good as the numeraire.

Matches last until either one partner dies or until an exogenous shock causes a separation. If a partner dies, the survivor becomes unemployed and begins searching for a new match. If the parties become separated, both must re-enter the labour market in search of employment. Therefore, the survival rate for a partnership is given by

8. We have chosen to work in discrete time due to some complexities that arise in continuous-time, overlapping-generations models with search costs. The continuous-time analogue of our model yields exactly the same conclusions as our discrete-time model and is available from the authors upon request.

$h=(1-d)^2(1-b)$. That is, the probability that a partnership survives is equal to the joint probability that both partners live and the job does not break up.

We do not allow agents to save so that all income is spent on the consumption good.[9] Employed workers earn income by producing and selling output, while unemployed workers earn nothing. For simplicity, we assume that each unit of X yields one unit of utility and that utility is separable in consumption and search effort. This implies that expected utility is equal to expected income minus search costs.

Finally, consider the search process. Each searcher can influence the probability of employment by altering search effort. We use e_i to denote the search intensity of agent i and assume that the cost of search, $c(e_i)$, is increasing and convex with $c(0)=c'(0)=0$. The search technology is introduced by assuming that the total number of new matches created (M) is an increasing and concave function of aggregate search effort ($E=\sum_{i\in S} e_i$ where S denotes the set of searchers) with $M(0)=0$ and $\lim_{E\to O} M'(E)=\infty$. In addition, M is bounded above by the cardinality of S. Assuming that all searchers are equally likely to find employment, the probability of any given individual finding a match is given by[10]

$$\pi_i(E)=\frac{2M(E)}{L-2Xh}. \tag{20}$$

Since each match consists of two workers, the numerator is equal to the number of vacancies filled in each period. The denominator equals the total number of searchers at the beginning of the period since hX jobs survive across periods and each job consists of a pair of matched workers.[11]

B. *Expected lifetime utility*

If we measure expected lifetime utility at the time that search costs are expended (at the beginning of the period) and if we assume that agents do not discount the future, then we have

$$V_E=\tfrac{1}{2}+hV_E+(1-d-h)V_U, \tag{21}$$

$$V_U=\pi_i\{\tfrac{1}{2}+hV_E+(1-d-h)V_U\}+(1-\pi_i)(1-d)V_U-c(e_i). \tag{22}$$

For a searcher, the probability of finding immediate employment is π_i, in which case current income is 1/2. In addition, once matched, the probability that the partnership survives to the next period is h (leading to V_E in the future) and the probability that the worker survives but loses his job is $(1-d-h)$ (leading to V_U in the future). The searcher remains unemployed and earns no current income with probability $(1-\pi_i)$. He survives to search again in the next period with probability $(1-d)$, and this leads to future income of V_U. Search costs are then subtracted to obtain V_U. Similar logic explains (21).

For future reference, we solve (21) and (22) to obtain an expression for the value of employment. Whenever a match occurs, the expected lifetime utility for each partner increases from V_U to V_E so that the total value of a match is $2[V_E-V_U]$. Since all jobless

9. In Appendix B of the working paper precursor to this article (Davidson, Martin, and Matusz (1989)) we provide an extension of our model that allows for savings and show that all of our results generalize to such a framework.

10. The assumption that all agents are equally likely to find a match might seem peculiar in a model in which search effort is endogenous. However, we employ this type of search technology for convenience alone. As should be clear from our analysis, none of our results depend on this particular search technology.

11. Note that the number of agents who begin the period as searchers, $L-2Xh$, is not equal to the number who end the period unemployed, $L-2X$.

agents are identical, we assume that they all choose the same search effort (e). We therefore omit the subscripts used to denote individual values for the remainder of the paper. Doing so allows us to write the value of a match as

$$2[V_E - V_U] = \frac{1 - \pi + 2c(e)}{1 - h(1 - \pi)} \tag{23}$$

where π is evaluated at $(L - 2Xh)e$. Note that the value of a match is decreasing in π—they are most valuable when they are hard to obtain.

C. *Steady states and the social surplus*

In a steady state the flows into and out of employment are equal. The number of matches destroyed each period is equal to $(1-h)X$ while the number of new matches created is M. The steady-state condition is therefore given by

$$M = (1-h)X. \tag{24}$$

We can now combine (20) and (24) to obtain a relationship between X and E that must hold in any symmetric steady state

$$\pi L = 2X[1 - h(1 - \pi)]. \tag{25}$$

In a market equilibrium, each unemployed worker chooses e_i to maximize V_U based on correct conjectures concerning the search effort of all other searchers. We could apply Bellman's Principle of Optimality and maximize (22) over e_i to obtain this equilibrium expression and close the model. However, in this paper we are not interested in examining the properties of equilibrium. Instead, we wish to show that a social surplus exists in *any* symmetric steady state—that is, for any given value of e. Therefore, we are now ready to move on.

To find conditions under which a social surplus exists, we now sum the expected lifetime utilities of all workers currently alive. At the beginning of any given period, there are $2Xh$ matched individuals while the remainder of the population is unemployed. Therefore, we have

$$V = 2XhV_E + (L - 2Xh)V_U = LV_U + 2Xh[V_E - V_U]. \tag{26}$$

If we solve (21) and (22) for V_E and V_U and then use (25) to substitute for L we obtain

$$V_U = \frac{X - (L - 2Xh)c(e)}{Ld} \tag{27}$$

which implies

$$\mathrm{V} = \frac{\mathrm{X} - (\mathrm{L} - 2\mathrm{Xh})\mathrm{c}(\mathrm{e})}{\mathrm{d}} + 2\mathrm{Xh}[\mathrm{V}_E - \mathrm{V}_U]. \tag{28}$$

Equation (28) is analogous to (1) in the introduction and (12) in the previous section. It verifies that the expected lifetime utility of the current generation consists of two components. The first, $[X - (L - 2Xh)c(e)]/d$, is the value of steady-state output produced during the lifetime of a representative agent (net of search costs). The second term, $2Xh[V_E - V_U]$, reflects the increase in expected utility (above what searchers expect to achieve) that can be attributed to the *durable* jobs held in steady state. This term also reflects the surplus value from employment. As indicated by (27), this surplus does not come at the expense of the newborns since they can still expect to earn their share of the output produced during their expected lifetime.[12] In contrast with the model of the previous section, this pure surplus exists for any $X>0$—no government action is required to produce it.

Like the minimum wage, a durable job transfers income across generations, and it is these transfers that create the surplus. Although the general argument is similar, there are some differences in the details. For example, in the minimum wage model it is the increase in the wage that enlarges the current generation's share of future output. Here, it is the higher level of steady-state employment that does the trick. In addition, in the minimum wage model, any increase in the wage reduces the current income of the newborns as the return to capital falls. However, they make up for this by earning more income in the future when they are employed. In fact, the gains and losses are always equal so that there is no change in their expected income. Here, if we compare two steady states with different employment levels, the newborns suffer when they are young in the high-employment case because they are crowded out of the labour market by the older workers (there are fewer vacancies). They recoup some of their losses as they age and take on the role of the mature generation since they will then possess a greater claim on future output. Unlike the minimum wage model, these two opposing forces need not be equal.

In determining whether they prefer the low- or high-employment steady state, the newborns compare the losses when they are young to the gains when they are old. Since all newborns begin life unemployed, their interests are reflected by V_U as given in (27). If net output is increasing (decreasing) in X, then the gains in old age dominate (are dominated by) the losses when they are young and the newborns prefer the high-employment (low-employment) steady state. When net output is maximized (call this output level X^*), these two opposing forces offset and a marginal change in employment has only second-order effects on the welfare of the newborns.

The interests of the current generation are reflected in V as given in (28). If employment is such that $X=X^*$ *and* if the surplus value from employment is increasing in X at this output level,[13] then this generation would gain from a marginal increase in employment. This follows from the fact that there would be a first-order increase in the surplus and only a second-order loss in the value of net output. Intuitively, at this level of employment a marginal increase in employment transfers income forward. The current generation receives a transfer from the newborns and they gain. Every other generation transfers income to the generation preceding them when they are young and receives an equal transfer when they are old from the generation succeeding them.

It is tempting to conclude at this point that a marginal increase in employment above the level that maximizes net output would be Pareto-improving. After all, there are first-order gains by the current generation and only second-order losses suffered by future

12. Tirole (1985) demonstrates that asset bubbles may appear in the equilibria of growing overlapping-generations economies. That is, the equilibrium value of an asset may be greater than its fundamental value. Since a job in our economy implies an increase in expected future earnings, we could think of each job as an asset. One might then be tempted to think of the surplus value as an asset bubble. However, this is not the case. As Tirole has shown (Proposition 1), for these asset bubbles to appear, the economy must grow at a rate faster than the rate of time preference. Since our economy does not grow (we have fixed factor supplies and constant technology), asset bubbles cannot be supported in equilibrium.

13. The surplus value from employment is given by $2Xh(V_E-V_U)$. If we use (24) to substitute for M in (20), it is easy to show that π is increasing in X. In addition, we noted on page 183 that V_E-V_U is decreasing in π. Therefore, as X increases, (V_E-V_U) decreases. This implies that $2Xh(V_E-V_U)$ could be increasing or decreasing in X. We investigated this issue in greater detail in our working paper (Davidson, Martin, and Matusz (1989)) and showed that for most empirically relevant values of the parameters, the surplus is increasing in X. Of course, even if $2Xh(V_E-V_U)$ is decreasing in X at X^*, X^* is still Pareto-dominated. The only difference is that we would need a marginal decrease in output to obtain the Pareto improvement.

generations. However, there are an infinite number of future generations that lose and only one current generation that gains. Therefore, while it is possible to rig policies that are Pareto-improving, one needs to be careful in choosing the policy in order to make sure that all losers are compensated sufficiently.[14]

The spartan nature of our model makes it difficult to find simple Pareto-improving policies. However, if we extend the model to allow for a safe asset, then there is a simple policy that always works. All that the government needs to do is tax away half of the current generation's first-order gain, invest the tax revenue in the safe asset, and use the return in each future period to provide the newborns with a payment that dominates the second-order loss. Since the current generation still receives a first-order gain, lump-sum taxation within the generation can be used to insure that everyone in that cohort is made better off. The result is a Pareto improvement. We provide such an extension in Appendix B.

4. EFFICIENCY WAGES

There is nothing special about the search model developed in the previous section. The surplus from employment exists in any dynamic model of unemployment in which $V_E \neq V_U$, jobs are durable, and workers are finitely-lived. To illustrate this point, we now demonstrate that the surplus exists in the Shapiro-Stiglitz efficiency-wage model once it is modified to allow for finite life.

In the Shapiro–Stiglitz model, utility is given by $w-e$ where w is the wage and e is effort expended on the job by employed workers. Employed workers choose either $e>0$ (a *fixed* positive level of effort) or $e=0$. Workers who shirk (choose $e=0$) earn an expected lifetime *labour* income of V_E^S while the non-shirking workers earn V_E^N. There is an exogenous probability of b that any given worker will be terminated and shirking workers face an additional probability of termination equal to q. Therefore, the expected lifetime labour incomes of shirking and non-shirking workers are given by

$$V_E^S = w + (1-d)[(b+q)V_U + (1-b-q)V_E^S], \tag{29}$$

$$V_E^N = (w-e) + (1-d)[bV_U + (1-b)V_E^N], \tag{30}$$

where V_U is the expected lifetime labour income for an unemployed worker. In each equation, expected lifetime labour income is equal to current income plus expected future labour income.

Workers shirk unless $V_E^N \geqq V_E^S$. Therefore, the wage satisfies the "no-shirk condition" if

$$w \geqq dV_U + \frac{e}{q}\left[q + \frac{1-h}{1-d}\right], \tag{31}$$

which must hold in equilibrium. Thus, all workers choose $e>0$ and $V_E = V_E^N$.

For unemployed workers, expected lifetime labour income is given by

$$V_U = \pi V_E + (1-\pi)(1-d)V_U. \tag{32}$$

14. This problem does not arise in models with discounting and population growth if the population grows at a rate greater than the discount rate. The reason for this is as follows. Let n denote the population growth rate and let ρ denote the discount rate. Then, if $n>\rho$, it can be shown that when X increases marginally above X^* the losses the newborn suffer when they are young are smaller than the gains they receive when they are old. Therefore, such an increase in employment generates first-order gains for all cohorts. It follows that X^* is Pareto dominated. It is in this sense that our results are strengthened when $n>\rho$ (see footnote 6).

That is, with probability π the worker finds a job and expects to earn V_E, and with probability $(1-\pi)(1-d)$ the worker fails to find a job and survives to seek employment again in the next period.

Solving (30) and (32) yields

$$V_E=\frac{(w-e)[d+\pi(1-d)]}{d[1-h(1-\pi)]}, \tag{33}$$

$$V_U=\frac{\pi(w-e)}{d[1-h(1-\pi)]}, \tag{34}$$

where h, the job survival rate, equals $(1-d)(1-b)$.

The employment probability, π, is equal to the ratio of job openings $(L_E(1-h))$ to job seekers $(L-L_Eh)$. Therefore,

$$\pi=\frac{L_E(1-h)}{L-L_Eh}. \tag{35}$$

Turn next to the production side of the economy. Assume that there is a single competitive firm that faces a concave production function represented by $F(L_E)$. Since the firm's per period profit equals $X-wL_E$, the expected lifetime earnings from capital are given by

$$V_K=\frac{X-wL_E}{d}. \tag{36}$$

We are now in a position to prove that a social surplus exists in any steady state. Aggregate expected lifetime income for the current generation is given by $V=hL_EV_E+(L-hL_E)V_U+V_K$, while a typical newborn expects to earn V_U from labour and V_K/L from his share of the firm's profits. Substituting from (33)–(36) yields

$$V=\frac{X-eL_E}{d}+hL_E(V_E-V_U), \tag{37}$$

$$V_U+\frac{V_K}{L}=\frac{X-eL_E}{dL}. \tag{38}$$

Equation (37) is analogous to (1), (12), and (28). It indicates that the aggregate expected income of the current generation exceeds the output produced during its lifetime. Equation (38) is analogous to (17) and (27) and indicates that each newborn expects to earn an equal share of the net output produced during his expected lifetime. Thus, the current generation's extra consumption does not come at the expense of the unborn—there is a pure surplus.

5. DISCUSSION

In this section, we compare our results to some previous work in the equilibrium unemployment and overlapping-generations literatures. In addition, we briefly discuss some of the difficulties involved in determining the optimal policy in this framework.

A. *Related Work*

One of our main results is that the current generation can consume more than the value of the net output produced during its expected lifetime without the extra consumption coming at the expense of future generations. As we noted in the introduction, this is reminiscent of Samuelson's (1958) seminal result that in an overlapping-generations framework a surplus could be generated by government programmes designed to borrow from future generations. In particular, he showed that transfers to the older generation could be financed by taxing the younger generation with the ultimate repayment postponed to the infinitely distant future. The competitive equilibrium therefore passes up an opportunity to increase the utility of the older generation (at no expense) and is sub-optimal.

In that our surplus is also the result of a transfer from the future to the present, these results are similar. However, there are important differences as well. Most importantly, no surplus is present in the competitive equilibrium in Samuelson's model, while a surplus is always present in our framework (provided that the employed and unemployed expect to earn different lifetime incomes). Moreover, while the surplus is created by an imaginative government policy (e.g. social security financed on a pay-as-you-go basis) in the consumption-loan model, it arises endogenously in models with a natural rate of unemployment. Finally, one of the implications of the surplus in our model is that the interests of current and future generations diverge. While this conflict is also present in most overlapping-generations models, it is due to the fact that the pattern of consumption varies with age and is in no way related to a surplus.

Another result from the OLG literature that is somewhat relevant is due to Blanchard (1985) who showed that in a model with an uncertain length of life, the probability of death drives a wedge between the individual's discount rate and the economy's. He then goes on to show that in a model with capital accumulation this difference in discount rates leads workers to save such that their wealth grows faster than the economy. In our model, a similar result emerges in that the discount factor for individuals is $(1-d)$, while the explicit interest rate for the economy is zero. In addition, although we do not allow agents to hold physical wealth, it is true that average expected lifetime consumption increases from $[(X-c(X))/Ld]$ at birth to a higher value as members of the cohort age. However, this result is not due to savings behaviour, rather it follows from the changing employment status of the representative agent over the lifespan of the generation.

In comparing our results with those in the literature on unemployment, we simplify matters by focusing on the search and efficiency wage literatures. A common theme in much of this work is that equilibrium is generally inefficient due to externalities inherent in either the search process (e.g. Diamond (1982, 1984), Mortensen (1982) or Pissarides (1984, 1990)) or the monitoring process (e.g. Shapiro and Stiglitz (1984)). This is often proved by demonstrating that an alternative feasible allocation exists that raises the value of steady-state output. Since almost all of these models are characterized by a single infinitely-lived generation, this procedure is entirely appropriate. However, if these models were extended to allow for death, our results indicate that this procedure would no longer be appropriate—the allocation that maximizes steady-state output would be Pareto-dominated by another feasible allocation. Two notable models of unemployment that *do* employ an overlapping-generation structure are Albrecht and Axell (1984) and Pissarides (1991). Since both models satisfy the criteria that are needed to generate a social surplus, our analysis suggests that the efficiency analysis carried out in these articles is incomplete.

B. *Policy*

What is the optimal policy in this setting? What policies are likely to be implemented? The answers to these questions are not straightforward. We have argued that policies aimed at maximizing the value of steady-state output are inefficient since they pass up the opportunity to achieve a Pareto improvement. However, it is difficult to say much more without making additional assumptions. Clearly, the policies that will be implemented will depend on the political process in place. Suppose, for example, that we make the reasonable assumptions that only those currently alive are allowed to vote and that the majority rules. Suppose further that we assume that individuals care only about their own expected lifetime incomes and that the employed outnumber the unemployed. Then, since the employed expect to earn V_E, policy will be targeted at increasing V_E. These policies may harm the unemployed, since their interests are represented by V_U. Of course, the fact that the employed and the unemployed have different interests is not a unique feature of our model—this result arises in almost any model of unemployment.

Now, suppose instead that policy is determined by some political leader who wishes to maximize the aggregate expected lifetime income of his constituency (i.e. the current generation). As we have shown above, V, the aggregate expected lifetime income of the current generation, consists of two terms—net output and the surplus value from employment. Therefore, the policy that would be implemented would take into account the social surplus. Such policies may, however, harm the unborn, since their interests lie only in net output. In fact, we have shown elsewhere (Davidson, Martin and Matusz (1989)) that the current generation typically prefers policies aimed at expanding employment beyond the level preferred by the unborn. However, once a cohort is born and matures (so that it takes on the role of the current generation) they will then favour policies that maximize V—in a sense, such policies possess a degree of time-consistency. This particular conflict of interest between current and future generations is unique to our model and is the direct result of the surplus value from employment.

Finally, suppose that policy is determined by a social planner who takes into account the interests of all agents—the employed, the unemployed, the living, and the unborn. The result would be an attempt to maximize some weighted sum of the value of net output and the surplus value from employment. Since the interests of the current generation enter into social welfare, *the surplus would matter*.

The manner in which the social surplus affects the optimal policy is beyond the scope of this paper. However, there is one important implication that we would like to point out. There is a substantial and growing literature in labour economics that concerns changes in the sectoral distribution of employment over time and its effect on earnings (see, for examples, the collection of papers in the February 1992 *Quarterly Journal of Economics*). Implicit in this literature is the idea that the sectoral distribution of output plays a role in determining economic welfare. Jobs in some sectors are perceived as "good jobs" (e.g. manufacturing) while others are viewed as "bad jobs" (e.g. minimum wage or low-paying service sector jobs). Recent shifts in employment from good to bad sectors are viewed as detrimental to the US economy and have led some to call for an American industrial policy aimed at encouraging employment in the good sectors. These arguments may be justified when externalities or distortions are present. But, in a competitive labour market, as Bulow and Summers (1986) have observed, "the claims of industrial policy advocates are difficult to understand. Competition equalizes the marginal productivities of all equivalent workers. There is no such thing as a good or a bad industry." In other words, in a non-distorted equilibrium the marginal job in each sector adds the same amount to national income so that a reallocation of employment cannot be beneficial.

This is not the case in our framework. In Davidson, Martin and Matusz (1991) we present a two-sector version of our search model and show that the surplus value from employment varies across sectors. Durable jobs that are difficult to obtain carry with them a larger social surplus than those that are transient and easy to obtain. This follows from the fact that the difference between what the employed and the unemployed expect to earn in a specific sector is tied to the turnover rates—$(V_E - V_U)$ is increasing in h, the job survival rate, and decreasing in π, the employment probability. Therefore, even if the marginal productivity of labour is equated across sectors, so that the marginal job in each sector contributes the same amount to net output, a reallocation of labour towards the high surplus value sector would increase the expected lifetime income of the current generation (V)—*the sectoral composition of output would matter*! It is important to note that this result does not arise because equilibrium is distorted (as in the efficiency wage model of Bulow and Summers). It arises as a direct result of the social surplus from employment and a difference in turnover rates across sectors. We explore this issue in much greater depth in Davidson, Martin and Matusz (1991).

APPENDIX A

In order to highlight the importance of death, we modify the basic minimum wage model of Section 2 to allow for infinitely-lived workers. If workers are assumed to live forever, we need to allow for discounting, otherwise expected utility would be unbounded. We use $\rho > 0$ to represent the discount rate. All other notation and assumptions remain the same as in the text.

The expected incomes of the employed and unemployed are now (the reader is reminded that these values are measured at the beginning of each period)

$$V_E = w + ar + \frac{1}{1+\rho}\{(1-b)V_E + bV_U\}, \tag{A.1}$$

$$V_U = \pi V_E + (1-\pi)\left[ar + \frac{1}{1+\rho}V_U\right]. \tag{A.2}$$

The probability of finding a job (π) can be obtained from the text by setting $d=1$ in (5). We obtain

$$\pi = \frac{\alpha b}{1-\alpha+\alpha b} < 1. \tag{A.3}$$

We can now substitute (A.3) into (A.1) and (A.2), and then solve for V_E and V_U. Doing so yields

$$V_E = \frac{1+\rho}{\rho}\left[\frac{\alpha b(1+\rho)+\rho(1-\alpha)}{\alpha b(1+\rho)+(\rho+b)(1-\alpha)}\right]w + \frac{1+\rho}{\rho}\alpha r, \tag{A.4}$$

$$V_U = \frac{1+\rho}{\rho}\left[\frac{\alpha b(1+\rho)}{\alpha b(1+\rho)+(\rho+b)(1-\alpha)}\right]w + \frac{1+\rho}{\rho}\alpha r. \tag{A.5}$$

By definition, $V = (1-b)L_E V_E + [L-(1-b)L_E]V_U$, where V is aggregate expected income. We can rewrite this expression to highlight the differential expected income due to employment: $V = LV_U + (1-b)L_E(V_E - V_U)$. Using (A.4) and (A.5), along with the fact that $\alpha L = K = L_E$, we have

$$LV_U = \frac{1+\rho}{\rho}(\phi w L_E + rK) \tag{A.6}$$

and

$$(1-b)L_E(V_E - V_U) = \frac{1+\rho}{\rho}(1-\phi)L_E w, \tag{A.7}$$

where

$$\phi = \frac{b(1+\rho)}{\alpha b(1+\rho)+(\rho+b)(1-\alpha)} \le 1, \quad \text{equality if } b=1. \tag{A.8}$$

From (A.6) and (A.7), it is easy to see that

$$V=\frac{1+\rho}{\rho}X, \tag{A.9}$$

and there is no surplus. Furthermore, using the fact that $r=1-w$, it is a simple matter to show that V_E is increasing in w, while V_U is decreasing in w. The employed benefit from the implementation of a minimum wage *at the expense* of the unemployed.

APPENDIX B

To demonstrate that a marginal increase in output above X^* can yield a Pareto improvement, we add capital to the search model of Section 3. Assume that each matched pair can rent capital from a frictionless market at a per unit price of r, and that the k_i units of capital rented by pair i allows them to produce $f(k_i)$ units of output. Capital does not depreciate (it can be thought of as land). As in our minimum wage model, all living agents own an equal share of the capital with a representing the *per capita* capital ownership. Finally, we assume that capital can be produced according to the following cost function

$$g=g(K), \tag{B.1}$$

where K is the total capital stock at the beginning of the period.

We consider steady states in which each unemployed worker expends e units of search effort and each matched pair hires (K/X) units of capital. With this modification, we can rewrite (21)-(23) as

$$V_E=\frac{1}{2}\left[f\left(\frac{K}{X}\right)-\frac{K}{X}r\right]+ar+hV_E+(1-d-h)V_U, \tag{B.2}$$

$$V_U=\pi\left[\frac{1}{2}\left(f\left(\frac{K}{X}\right)-\frac{K}{X}r\right)+ar+hV_E+(1-d-h)V_U\right]+(1-\pi)(1-d)V_U-c(e), \tag{B.3}$$

$$2[V_E-V_U]=\frac{(1-\pi)f(K/X)+2c(e)}{1-h(1-\pi)}. \tag{B.4}$$

Solving for V_U we obtain

$$V_U=\frac{Xf(K/X)-(L-2hX)c(e)}{dL}, \tag{B.5}$$

which indicates that a representative newborn still expects to consume an appropriately weighted share of the net output produced during his lifetime. As in the text, let $X^*(K)$ denote the value of X that maximizes this expression given K.

Finally, if we let K_n denote the capital stock at the end of period, then (28), the aggregate expected lifetime of the current generation becomes

$$V=\frac{Xf(K_n/X)-(L-2hX)c(e)}{d}+2hX(V_E-V_U)-[g(K_n)-g(K)]. \tag{B.6}$$

The first two terms reflect the net output produced during an average lifetime and the surplus value from employment. The third term reflects the added cost of any new capital created by the current generation. In a steady state, $K_n=K$, no new capital is created, and no costs are incurred.

We assume that at the beginning of each period the current generation decides whether or not to expand the capital stock. If they decide to do so, they set K_n such that $(\partial V/\partial K_n)=0$. Let $K^*(X)$ solve this expression given X and let $(K^*, X^*)\equiv(K^*(X^*), X^*(K^*))$.

Now, suppose that the economy is in a steady state with $X=X^*$ and $K=K^*$. Consider the impact of a marginal increase in both X and K. Because the capital lasts forever, this can be seen as an exchange between the current and all future generations. Level sets of V_U and V (as given in (B.5) and (B.6)) are depicted in Figure 1. Since V_U is proportional to net output and since the current generation incurs the cost of increasing K, the slope of the newborns' indifference curve approaches infinity at (K^*, X^*)—that is, if we take the limit as (X, K) approaches (X^*, K^*) we have

$$\frac{\partial V_U(X,K)/\partial K_n}{\partial V_U(X,K)/\partial X}=\infty.$$

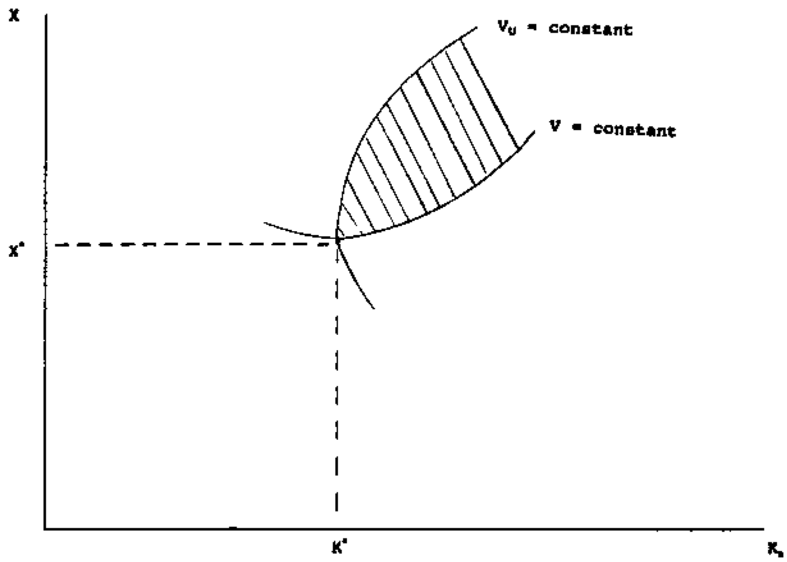

FIGURE 1

For the current generation, if we take the limit as (X, K) approaches (X^*, K^*), we have

$$\frac{\partial V(X, K)/\partial K_n}{\partial V(X, K)/\partial X} = 0,$$

by the definition of K^*. Therefore, any allocation in the shaded area of Figure 1 benefits both generations (all of these allocations imply greater output). Intuitively, the increase in X creates a first-order gain for the current generation while the increase in K generates only second-order losses. Therefore, the current generation gains. For the newborns, the increase in K generates first-order gains (since they do not incur the cost of increasing K) while the increase in X produces second-order losses. Therefore, the newborns gain as well. This proposal could be implemented by using job subsidies to increase e (and hence X) while taxing the current generation a small amount (so as not to tax away all of their first-order gain). The tax revenue could then be used by the government to purchase the new capital.

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