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EXCESS CAPACITY AND COLLUSION*

BY CARL DAVIDSON AND RAYMOND DENECKERE¹

In this paper, we analyze a restricted class of equilibria in the dynamic model of Benoit and Krishna (1987) in which firms choose their scale of operation before engaging in a repeated game of price competition. Benoit and Krishna established that all firms carry excess capacity in all collusive equilibria. As we are interested in the relationship between excess capacity and collusion in price, we examine equilibria in which firms tacitly collude in price but not in investment decisions. If we further restrict attention to equilibria that are Pareto undominated within this class of "semi-collusive" equilibria, we find that capacity levels and collusion both increase if either interest rates or the cost of capacity fall. Increases in the interest rate reduce the capitalized value of the losses due to retaliation, thereby making it more difficult to support collusion. As excess capacity is used to support collusive outcomes, less capacity is needed if less collusion results.

1. INTRODUCTION

Recently, there has been a tremendous resurgence of interest in oligopoly theory. The impetus for this resurgence stems from the consideration of dynamic aspects of oligopolistic markets. Two particular topics that have attracted a great deal of attention are the ability of firms to collude in a repeated game setting and the role that the timing of decisions—especially investment decisions—plays in determining oligopolistic outcomes. Friedman (1971) demonstrated that, provided the discount factor is not too low (and the detection lags are not too long), collusive output levels can be supported by a threat of all industry members to permanently revert to the static Nash equilibrium if cheating is ever detected. Subsequent work by Green and Porter (1984), Abreu (1986) and Abreu et al. (1986) modified Friedman's analysis by altering the duration of punishments, introducing uncertainty and by developing credible threats that are more appealing than his "grim trigger strategies," either because they accomplish the same objective with less punishment or because they support more collusion. In addition, Brock and Scheinkman (1985) complemented Friedman's analysis by examining the stability of collusive agreements in a specific example of a price-setting game with capacity constraints (rather than a quantity-setting game). In all of the above papers, the capital stock is given exogenously at the outset of the game; the choice of the scale of operation is thus not modelled explicitly.

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Recent papers by Spence (1977), Dixit (1980), Eaton and Lipsey (1981), Gelman and Salop (1983), Kreps and Scheinkman (1983), and Fudenberg and Tirole (1983) study the influence of the timing of investment decisions on equilibrium outcomes in oligopolistic markets. Dixit, for example, shows that if an incumbent can choose his capital stock before a potential entrant can commit its resources, then by installing a large enough plant he may be able to deter entry. In a similar vein, Kreps and Scheinkman emphasize the importance of precommitment by showing that when firms can choose their capital levels before competing in prices, the outcome of the game will (under certain conditions) be identical to the Cournot outcome. In other words, the Cournot equilibrium can be viewed as the result of price competition between firms provided that prices are chosen after plant sizes. Note that in the above two models the authors assume that the initial competition is conducted through capacity (or scale of operation) while competition in prices and/or output follows, and that collusion does not take place. These features are shared by other papers dealing with the timing of investment decisions.

Recently, Benoit and Krishna (1987) presented a model that integrates the essential ideas of both strands of the literature. In their model, firms first choose a scale of operation (capacity limit), and then engage in an infinitely repeated game of price competition.² Firms are allowed to collude in both price and capacity. Collusion in capacity is accomplished by allowing firms to threaten (through their subsequent pricing policies) to retaliate against deviant capacity installments. Such threats may induce firms to restrict capacity, and thus provide the industry with an additional avenue for collusion. The main goal of Benoit and Krishna's paper is to find properties shared by all equilibria of their model. One such feature concerns excess capacity. In particular, they demonstrated that in all equilibria (except ones that mimic the Cournot-Nash outcome) firms carry excess capacity. This excess capacity allows firms to support prices above the Cournot-Nash level.

In this paper, we analyze a subset of equilibria of the Benoit-Krishna model.³ We are interested in the relationship between the level of excess capacity and the degree of price collusion that can be sustained in a market. We therefore restrict attention to a particular class of equilibria and distinguish between different types of equilibria within this class (based on their level of collusion). Specifically, we do not allow firms to collude in capacity. We assume that any threat of a price war following deviant capacity choices will be renegotiated after firms have made their investment decisions. More precisely, we assume that after *any* history of capacity choices, firms charge the maximum price that can be supported in a collusive agreement.⁴ The agreement is enforced by a threat (by all industry members) to

² The Benoit-Krishna model is a synthesis of Kreps and Scheinkman (1983) and Brock and Scheinkman (1985). It extends the Brock and Scheinkman paper by allowing capacity constraints to be determined endogenously and it extends the Kreps and Scheinkman analysis by allowing the price game to be repeated infinitely so that collusive outcomes can be supported in equilibrium.

³ Throughout this paper we refer to the model as the "Benoit-Krishna model" due to the complete analysis of equilibrium provided by Benoit and Krishna (1987). However, such a model was independently introduced and analyzed in Davidson and Deneckere (1984).

⁴ We do not literally mean the maximum price, but rather the highest sustainable price in the (closed) interval below the monopoly price.

permanently revert to the static-Nash equilibrium as soon as anyone is caught cheating. Thus, when a firm contemplates deviating from the agreement, it weighs the immediate gains from cheating against the capitalized value of future losses due to retaliation. The maximum sustainable price is defined to be the price that maximizes the "cartel welfare function" subject to the constraint that all firms find it optimal to abide by the agreement.

Given that firms cannot collude in capacity even though they may be colluding in price, we refer to our restricted set of equilibria as "semi-collusive" equilibria. The major focus of our paper is on the effects of changes in the interest rates or the cost of capacity on the relationship between the level of excess capacity and the degree of collusion that can be sustained in equilibrium. Our restriction is motivated by the observation that firms frequently compete in some strategic dimensions, while choosing to cooperate in others (Scherer 1980; Brander and Harris 1984; Fershtman and Muller 1986). Apparently, firms find it much more difficult to coordinate decisions regarding long-run variables such as investment, advertising, and/or research and development expenditures than decisions concerning short-run variables such as price and/or output. Several examples of industries in which firms colluded in price and/or output but not in investment are cited in Scherer (1980, pp. 370-71), and include the nitrogenous fertilizer and synthetic fibers industries during the 1960s as well as the plastics and aluminum industries during the 1950s. Furthermore, it is well-known that even in cases of overt collusion (such as the German cement cartel in the 1920s and 1930s, or the Texas oil industry in the 1930s) firms find it exceedingly difficult to collude in capacities (Brander and Harris 1984).⁵

Settings in which rivals collude in some but not all dimensions have been labelled "mixed games" by Brander and Harris (1984). In spite of their noted empirical relevance, mixed games have not received much attention in the theoretical literature. While we make no attempt in this paper to explain why firms do not coordinate investment decisions,⁶ we believe the widespread observance of this phenomenon warrants an investigation of the nature of the equilibria that result in such a setting.

Formally, we characterize the subgame perfect Nash equilibria of a two-stage game in which firms first choose capacity levels and then, in a second stage, the maximum price that can be sustained in a tacitly collusive agreement. The semi-collusive equilibria are calculated by first solving for the maximum sustainable price and profit levels as a function of industry capacity levels. The reduced form payoff functions then allow us to determine the Nash equilibria in capacities. As our semi-collusive equilibria are contained in the full set of equilibria studied by Benoit and Krishna, excess capacity is carried in all equilibria (other than those in which

⁵ In his description of the German cartels, Scherer (1980, pp. 370-71) writes: "In Germany during the 1920s and 1930s, shares were allocated on the basis of production capacity. Cartel members therefore raced to increase their sales quotas by building more capacity. . . . Even when market shares are not linked formally to capacity, a cartel member's bargaining power depends upon its fighting reserves—the amount of output it can dump on the market, depressing the market, if others hold out for unacceptably high quotas."

⁶ Casual arguments have included the observation that long-run variables are inherently more difficult to monitor than short-run variables (Fershtman and Muller 1986).

firms produce the static-Nash equilibrium quantities). As emphasized by Benoit and Krishna, this excess capacity is necessary to punish deviations from the collusive scheme, were these to occur. Furthermore, all semi-collusive equilibria involve capacities at or above their static Cournot-Nash equivalent.

The type and number of equilibria in the model depend upon two critical parameters—the cost of capacity and the discount rate. All equilibria are symmetric and are characterized by the level of capacity firms possess, the price they charge and the output they produce. There are basically three types of semi-collusive equilibria: (a) equilibria in which firms carry considerable excess capacity and charge a price which a monopolist with zero costs of capacity would choose (henceforth referred to as unconstrained semi-collusive equilibria (USE)), (b) equilibria in which excess capacity is not sufficient to support this monopoly price but is large enough to support some price above the level that would be charged in the static Nash equilibrium (constrained semi-collusive equilibria (CSE)), and (c) equilibria in which firms carry no excess capacity. In the latter type of equilibrium the price and output levels in each period coincide with their static Cournot-Nash equivalents. We refer to those equilibria as noncollusive equilibria (NCE). There is at most one USE and at most one NCE, but neither need exist. In contrast, there is generally a continuum of CSE. Both the USE and NCE may simultaneously exist in addition to the CSE. If, however, attention is restricted to the set of semi-collusive equilibria that are not Pareto dominated (by any other semi-collusive equilibrium), then uniqueness obtains for almost all parameter values.⁷

Exogenous changes in the cost of capacity and the interest rate affect the semi-collusive price level. When capital is relatively cheap firms find it optimal to carry a great deal of excess capacity in order to deter cheating. In this case, an USE exists. This is also true if the interest rate is low since even minor threats of retaliation would then deter players from cheating, so that the monopoly price is sustainable. In fact, when capacity is cheap or the interest rate low, the USE is the unique equilibrium of the two-stage game. Increasing the cost of capacity or the interest rate creates equilibria of the CSE and NCE variety. In order to perform meaningful comparative statics, we restrict our analysis to the set of semi-collusive equilibria that are Pareto undominated by other semi-collusive equilibria (see Section 5). An increase in the cost of capacity decreases a firm's willingness to expand, while an increase in the interest rate leads it to discount the future more heavily. Thus, increasing either parameter lowers the degree of collusion that can be sustained in equilibrium (in moving us from an USE to a CSE). As the cost of capacity or the interest rate rise further, the level of collusion will continue to fall until the equilibrium becomes noncollusive. We thus find that decreases in the level of collusion are always accompanied by decreases in the amount of excess capacity carried by the industry.

⁷ Interestingly, in an otherwise apparently unrelated paper on strategic entry deterrence, Güibert and Vives (1986) developed a model which, at times, is also characterized by the coexistence of several types of equilibria (the types refer to whether or not the incumbent oligopolists deter entry). As in our model, these equilibria can be Pareto ranked. Moreover, it is also typically the case that if an entry preventing equilibrium exists, a continuum of such equilibria is actually present (as with our CSE).

The paper divides into five additional sections. In Section 2 we compare our results with the theoretical and empirical literature on the relationship between excess capacity and collusion. In Section 3 we present the model and define equilibrium. In Section 4, we solve for the equilibrium in the price subgame (with fixed capacities), and in Section 5 we discuss the equilibrium of the full game. We offer some concluding remarks in Section 6.

2. EXCESS CAPACITY AND COLLUSION

In our model increases in the level of collusion (due to changes in an exogenous parameter such as the interest rate) are always accompanied by increases in the levels of industry capacity and excess capacity. This is somewhat surprising since one of the well-known tenets of traditional oligopoly theory holds that greater levels of excess capacity weaken collusive agreements. Thus, the traditional view is that the level of excess capacity and the level of collusion are negatively correlated. It is argued that firms which carry a great deal of excess capacity have a strong incentive to cheat because they can capture a large share of the market by undercutting the collusive price. Firms with little or no excess capacity have no incentive to undercut since it is technologically infeasible (or extremely costly) for them to increase production. Thus, as the amount of excess capacity grows any collusive agreement is weakened.⁸ This argument, however, is incomplete: it ignores the effect of excess capacity on the ability of firms to retaliate when cheating occurs. After all, when the level of excess capacity is substantial the threat of retaliation looms large in the eyes of a potential cheater since firms can (and will) easily dump a large amount of output on the market to punish any chiseler. Similarly, when the level of excess capacity is relatively small a cheater need not worry very much about retaliation since the industry cannot cheaply expand production by any significant amount. Our results indicate that the retaliation effect tends to dominate the traditional chiseling effect and that excess capacity plays a prominent role in supporting collusive agreements. In fact, greater excess capacity brought about by a lower cost of capital allows firms to support more collusive prices. Therefore, our model predicts a positive correlation between the levels of excess capacity and collusion.⁹

The empirical evidence on the correlation between excess capacity and collusion is weak.¹⁰ At best one can say that there exist conflicting views and that most of the evidence cited in support of the traditional wisdom derives from case studies. The rayon, cement and heavy electrical equipment industries are examples of industries in which collusive agreements broke down in the presence of high levels of excess capacity.¹¹ In each of these instances, however, excess capacity arose because a sudden reduction in demand made it difficult for firms to earn profits even at the

⁸ See Scherer (1980), especially pp. 209-211.

⁹ While Benoit and Krishna (1987) emphasize the equilibrium coexistence of excess capacity and collusion, their model offers no specific prediction regarding the correlation between the levels of excess capacity and collusion.

¹⁰ See, for example, Esposito and Esposito (1974) and Mann, Meehan and Ramsey (1979).

¹¹ See Scherer (1980), pp. 209-211.

collusive price. Producers began to cut prices in the hope of surviving the recession by driving others out of the market. We believe that it was the decline in demand which led to overcapacity and price wars, and that any conclusion that excess capacity resulted in the dissolution of a collusive agreement is unwarranted.

As an example of an industry in which behavior is consistent with the predictions of our model we point to the steel industry in the 1950s and early 1960s. In this industry capacity utilization was rarely above 85 percent and often went below 75 percent.¹² Yet, prices remained high during this period in spite of changes in demand. It is also well-known that members of OPEC carried high levels of excess capacity during the time period that the cartel was strongest.¹³ Perhaps the most convincing piece of evidence in favor of our model concerns the United States' primary aluminum ingot market in the mid-1950s and 1960s. For this industry, Rosenbaum (1985) presents evidence that the price-cost margin was positively and significantly related to industry excess capacity (as a percentage of total capacity).

3. THE MODEL

Consider a market shielded from entry in which two firms produce a homogeneous product and engage in the following two-stage infinite horizon noncooperative game: in stage one (at time zero) each player simultaneously and independently purchases and installs capacity at a cost of c per unit. Capacity is infinitely lived, does not depreciate and can only be bought at time zero. In stage two (time periods one and beyond) firms compete in prices and produce output to order.¹⁴ We assume that no plant can be pushed beyond its capacity limits. Capacity thus serves as a proxy for the scale of production by placing an upper bound on any firm's output level. Throughout this paper we assume that the industry cannot collude in capacity even though it may be colluding in price.

It is well-known that in static or finite horizon models in which the component games have a unique Nash equilibrium, collusive outcomes cannot emerge in equilibrium as the result of a noncooperative game played by profit maximizing firms.¹⁵ The basic insight of the literature on repeated games is that if a market situation is repeated infinitely, the industry may settle at a collusive price even if firms are not explicitly colluding. Thus, in order to ensure that collusive outcomes may arise, we assume that the price game is repeated infinitely. In addition, since we are interested in the relationship between excess capacity and the degree of collusion, we assume that tacit collusion is the norm and that firms charge the maximum price sustainable in any collusive agreement.

The types of strategies that support collusion in supergames were sketched briefly in the introduction. In this paper we restrict attention to collusive agree-

¹² See the U.S. Congress, Senate Subcommittee on Antitrust and Monopoly Report No. 1387 (1958), pp. 45-51, 81-85.

¹³ See Scherer (1980).

¹⁴ The component games of the ensuing supergame coincide with Edgeworth's price-setting games with capacity constraints. Games of this type have been analyzed in detail in Kreps and Scheinkman (1983), Allen and Hellwig (1984), Davidson and Deneckere (1986), and Osborne and Pitchik (1986).

¹⁵ See, for example, Friedman (1977) and Benoit and Krishna (1984).

ments enforced by "grim trigger strategies." These strategies specify that firms remain at the collusive point unless someone cheats. If at any time anyone is detected cheating, players revert to the static Nash equilibrium and remain there forever.¹⁶ Firms will cheat if and only if their immediate gains from cheating dominate the capitalized value of losses due to retaliation. We assume that when the industry chooses a collusive price vector it is aware of the problems inherent in maintaining a collusive agreement. Thus, it always chooses the price vector that maximizes the "cartel welfare function" subject to the constraint that no cheating is ever induced.

To summarize and formalize the model presented thus far let π_i^c denote the per period profits earned by firm i at the cartel point (p_1, p_2) ; π_i^{ch} the per period profits earned by firm i when cheating optimally against (p_1, p_2) ; π_i^N the per period profits earned by firm i in the static Nash equilibrium, and r the interest rate. The net gains from cheating are given by:

$$(1) \quad Z_i = (\pi_i^{ch} - \pi_i^c) - \frac{1}{r} (\pi_i^c - \pi_i^N).$$

(In (1) and below, we suppress the arguments p_1, p_2, K_1, K_2 and r of Z_i , and the arguments K_1, K_2 and r of Ω .) Firm i cheats if $Z_i > 0$. Let Ω denote the set of prices that can be supported in a semi-collusive agreement:

$$(2) \quad \Omega = \{(p_1, p_2): Z_1 \leq 0 \text{ and } Z_2 \leq 0\}.$$

Finally, if we let $F(\pi_1^c, \pi_2^c)$ denote the cartel welfare function (with $F_1 > 0, F_2 > 0$) then the optimal sustainable price vector is given by the solution to the following maximization problem:

$$(3) \quad \max_{(p_1, p_2)} F(\pi_1^c, \pi_2^c), \quad \text{subject to } (p_1, p_2) \in \Omega.$$

The solution to (3) depends on the capacities chosen by firms in the first stage of the game. Let $\bar{\pi}_1^c(K_1, K_2)$ and $\bar{\pi}_2^c(K_1, K_2)$ represent the cartel profits evaluated at

¹⁶ Abreu (1983) has shown that in quantity-setting games firms may do better (i.e., support more collusion) when they use strategies other than grim trigger strategies. He does so by proving that an optimal punishment scheme exists (optimal in the sense that it supports the highest level of collusive profits), and that such a scheme requires firms to produce above the Cournot-Nash output level when punishing. Although Abreu's punishments seem attractive, they are quite difficult to characterize because in asymmetric games, they require nonstationary punishment paths (the two-phase punishment schemes which are easily computed for symmetric games work only if the discount factor is sufficiently high). Thus, it is highly unlikely that such strategies would ever be implemented in the real world. On the other hand, standard trigger strategies require only simple calculations and are easily understood by industry participants. It is more readily imagined that firms will use these simple punishments to support tacit agreements.

There are two other reasons why we make use of grim trigger strategies in our analysis. (i) It is easily proven that our punishments are optimal over regions (a) and (b) of Theorem 4. Thus, our analysis differs from Abreu's only over regions (c) and (d). In particular, in symmetric equilibria the punishments chosen will be optimal. (ii) Abreu's proofs depend on the continuity of payoff functions. Since payoff functions are discontinuous in price games it is unclear whether his results generalize.

the price vector that solves (3) and let $p_1^c(K_1, K_2)$ and $p_2^c(K_1, K_2)$ denote those prices. We are now in a position to define equilibrium in the two-stage game (for a fixed value of the interest rate r):

DEFINITION. $(K_1^*, K_2^*, p_1^*, p_2^*)$ is a semi-collusive equilibrium if:

- (a) $(p_1^*, p_2^*) = (p_1^c(K_1^*, K_2^*), p_2^c(K_1^*, K_2^*))$, and
 (b) $\bar{\pi}_1^c(K_1^*, K_2^*) - cK_1^* \geq \bar{\pi}_1^c(K_1, K_2^*) - cK_1$, for all K_1
 $\bar{\pi}_2^c(K_1^*, K_2^*) - cK_2^* \geq \bar{\pi}_2^c(K_1^*, K_2)$, for all K_2 .

(a) simply states that p_1^* and p_2^* solve (3) given K_1^* and K_2^* . The conditions in (b) guarantee that (K_1^*, K_2^*) constitutes a Nash equilibrium in capacities, given the subsequent pricing behavior.

4. EQUILIBRIUM IN THE PRICE SUBGAMES

In this section we compute the price vector that solves (3), as a function of the capacities in the industry. Let $D(p)$ denote the market demand curve and $P(x)$ its inverse. $P(0)$, the choke price, is assumed finite. In addition, we assume that $P(x)$ is strictly positive on some bounded interval $[0, \bar{x}]$, on which it is twice continuously differentiable and strictly decreasing (for $x \geq \bar{x}$, $P(x) \equiv 0$). Moreover, we assume that the revenue function ($xP(x)$) is single peaked, attaining a unique maximum at x_m and that this function is strictly concave on $[x_m, \bar{x}]$. Each firm can produce output at zero cost as long as capacity is not exceeded. These assumptions imply the existence of a unique pure-strategy quantity-setting equilibrium and allow us to characterize, in Theorem 1, the (possibly mixed strategy) price-setting equilibria that serve as a threat point for the collusive agreements.

In price-setting games firms may choose to charge different prices. If they do, we assume that customers first buy from the cheaper supplier. When the lowest priced supplier cannot satisfy all demand at that price, some customers will be left for the remaining firm. How much this firm will actually sell depends upon the pool of customers that remains to be served. We make the following simplifying assumption: the low priced firm serves the consumers with the highest reservation prices. Thus, if $p_i < p_j$, for some $i \in \{1, 2\}$ and $j \neq i$, firm j faces a contingent demand of:

$$(4) \quad q(p_j) = \max(0, D(p_j) - K_i),$$

and earns the following profits

$$\pi_j(p_1, p_2 | K_1, K_2) = p_j \min[\max(0, D(p_j) - K_i), K_j].$$

Firm i 's profits are given by

$$\pi_i(p_1, p_2 | K_1, K_2) = p_i \min(D(p_i), K_i).$$

When $p_1 = p_2$, firms share the market in some appropriate fashion (to be made precise below).

In order to solve program (3), we must calculate π^c , π^{ch} and π^N for both firms. This task is simplified by Theorem 1, which characterizes the static Nash equilibrium, and by Theorem 2, which proves that in any semi-collusive agreement (regardless of the capacity history), firms must charge identical prices. First, we must introduce some additional notation. Let $B(x) = \max_p [p(D(p) - x)]$, and let $v_i(K_1, K_2)$ denote firm i 's minmax payoff. That is,

$$v_i(K_1, K_2) = \inf_{p_j} \sup_{p_i} \pi_i(p_1, p_2 | K_1, K_2), \quad \text{for } i = 1, 2, j \neq i.$$

Also, define $R(x) = \arg \max_z \{zP(z + x)\}$ so that $R(x)$ is the Cournot best reply function. We can now state a theorem which provides the necessary information concerning the static-Nash equilibrium (observe that firms may use mixed strategies in equilibrium).

THEOREM 1. *For each pair (K_1, K_2) with $K_1 \leq K_2$, there exists a unique static Nash equilibrium in mixed strategies with equilibrium payoffs $\pi_i^N(K_1, K_2)$:*

(a) *If $K_2 \leq R(K_1)$, the equilibrium is in pure strategies, and:*

$$p_i = P(K_1 + K_2), \quad \pi_i^N(K_1, K_2) = K_i P(K_1 + K_2), \quad \text{for } i = 1, 2.$$

(b) *If $K_2 \geq R(K_1)$ and $K_1 < D(0)$, the equilibrium is in (nondegenerate) mixed strategies, and*

$$\begin{aligned} \pi_2^N(K_1, K_2) &= B(K_1), \\ \pi_1^N(K_1, K_2) &\geq K_1 B(K_1) / K_2. \end{aligned}$$

(c) *If $K_1 \geq D(0)$, the equilibrium is in pure strategies, and*

$$p_i = P(K_1 + K_2), \quad \pi_i^N(K_1 + K_2) = 0, \quad \text{for } i = 1, 2.$$

PROOF. The characterization of the equilibria, and the uniqueness of equilibrium profits is due to Kreps and Scheinkman (1983). Osborne and Pitchik (1986) showed that the equilibrium itself is unique under the present (more general) demand assumptions. □

We are now ready to establish the following result.

LEMMA 1. *Let p_i^c denote the price charged by firm i in a semi-collusive agreement after any history of capacity choices. Then $p_i^c < p_j^c$ for some $i \in \{1, 2\}$ and $j \neq i$ implies $\pi_i^c \leq \pi_j^N$.*

PROOF. If $p_1^c < p_2^c$ then firm two earns at most his minmax profits:

$$\begin{aligned} \pi_2^c &= \pi_2(p_1^c, p_2^c | K_1, K_2) \leq \sup_{p_2 > p_1^c} \pi_2(p_1^c, p_2 | K_1, K_2) \\ &\leq \sup_{p_2 > 0} \pi_2(0, p_2 | K_1, K_2) = v_2(K_1, K_2). \end{aligned}$$

Since in any mixed strategy Nash equilibrium, $\pi_2^N \geq v_2$ (Moulin, 1982, p. 45), the desired result follows. The argument for the case where $p_2^c < p_1^c$ is analogous. \square

We can now state:

THEOREM 2. *Let (K_1, K_2) be any pair of initial capacity choices, and let r be the interest rate. Then if $(p_1, p_2) \in \Omega(K_1, K_2, r)$, it follows that $p_1 = p_2$.¹⁷*

PROOF. Suppose to the contrary that $(p_i, p_j) \in \Omega$ satisfies $p_i < p_j$ for $i = 1$ or $2, j \neq i$. From Lemma 1, we have $\pi_j^c \leq \pi_j^N$. Since Z_j is nonpositive by assumption, we must have $\pi_j^{ch} = \pi_j^c$. Now if $\pi_i^c = \pi_i^N$, we will reach our desired contradiction since (i) $\pi_i^{ch} > \pi_i^c$ would imply $Z_i > 0$, and (ii) $\pi_i^{ch} = \pi_i^c$ (combined with $\pi_j^{ch} = \pi_j^c$) would imply that (p_1, p_2) is a pure strategy static Nash equilibrium and hence that we are in region (a) or (c) of Theorem 1, where $p_1 = p_2$. Thus, $\pi_i^c > \pi_i^N$.

To complete the proof let p denote the lowest price in the support of the Bertrand-Nash equilibrium distributions implicit in Theorem 1. Kreps and Scheinkman show that

$$\pi_j^N = \underline{p} \min(D(\underline{p}), K_j) \quad \text{and} \quad \pi_i^N = \underline{p} \min(D(\underline{p}), K_i).$$

Since $\pi_i^c > \pi_i^N$, it must be that $p_i > p$. Firm j can cheat on the agreement by charging a price \hat{p} such that $\underline{p} < \hat{p} < p_i$. This yields:

$$\pi_j^{ch} \geq \hat{p} \min(D(\hat{p}), K_j) > \underline{p} \min(D(\underline{p}), K_j) = \pi_j^N$$

contradicting $\pi_j^{ch} = \pi_j^c \leq \pi_j^N$. \square

Our consumer allocation rule (see equation (4) above) did not specify how market shares are determined when firms charge identical prices. The most natural assumption is that consumers sort in such a way that each firm's sales volume is proportional to the size of its plant. However, this sharing rule loses its intuitive appeal when plants become too large. For example, when each firm has a plant large enough to serve the entire market, it seems natural to assume that sales become independent of capacities. Also, in case (b) of Theorem 1 Nash profits and prices are independent of K_2 , and thus there is no real sense in which further increasing K_2 makes firm 2 any larger. In order to adequately deal with these problems, we suggest a slightly different sharing rule:

$$(5) \quad S_i = \frac{\min(K_i, \bar{x})}{\min(K_j, \bar{x}) + \min(K_i, \bar{x})},$$

¹⁷ It is a corollary of Benoit and Krishna (1987, Lemma 6) that along any stationary equilibrium path, firms must be charging identical prices. Theorem 2 makes a stronger claim, however, by also proving a similar result for nonequilibrium capacity choices. The distinction is significant since it is the equal price result that allows us to fairly easily describe the structure of $\Omega(K_1, K_2, r)$, and compute $p^c(K_1, K_2, r)$ in Theorem 3, below.

where S_i denotes firm i 's market share, and $j \neq i$. This sharing rule has two nice properties. First, sales are proportional to capacity when capacity matters. There is a significant body of evidence that cartels use such a rule in setting output quotas (see Brander and Harris 1984, and Osborne and Pitchik 1987). Secondly, when a firm becomes large enough to supply the entire market, market shares become independent of that firm's capacity.¹⁸

The use of sharing rule (5) and the fact that both firms must charge the same price at any point in Ω greatly simplifies the characterization of the solution to program (3). First of all, observe that for any p in Ω firms' profits (for $i = 1, 2$) are given by

$$(6) \quad \pi_i^c(K_1, K_2, p) = p \min (S_i D(p), K_i).$$

Both profit functions are increasing over the interval $[0, p^m]$ and decreasing over the interval $[p^m, P(0)]$, where $p^m = \max \{P(x^m), P(K_1 + K_2)\}$ is the price that a monopolist with capacity $K_1 + K_2$ and no cost of production would charge if capacity costs were sunk. Because these functions are single-peaked and reach their maximum at the same price p^m , the solution to program (3) is independent of $F(\cdot, \cdot)$. Moreover, since optimal cheating is accomplished by undercutting the semi-collusive price by an arbitrarily small amount, cheating profits are given by

$$(7) \quad \pi_i^{ch}(K_1, K_2, p) = \sup_{y < p} [y \min (D(y), K_i)] = p \min (D(p), K_i)$$

for $i = 1, 2$. Equations (5), (6), (7) and Theorem 1 provide the necessary information to describe Ω , the set of sustainable prices, for any given value of K_1, K_2 and r . If Ω is empty, we assume that firms resort to randomization, and revert to the static Nash equilibrium described in Theorem 1. If $p^m \in \Omega$, then p^m solves (3). This is because both π_1^c and π_2^c attain their maximum at $p^c = p^m$. Finally, if Ω is nonempty and $p^m \notin \Omega$, Ω will consist of an interval to the left of p^m (when p^m is not sustainable, then no price above p^m is sustainable either). The solution to program (3) then coincides with the right endpoint of this interval. We are now in a position to describe the equilibrium in the price subgames.¹⁹

THEOREM 3. *For each (K_1, K_2) with $K_1 \leq K_2$ and for every $F(\cdot, \cdot)$ satisfying the property $F_1 > 0, F_2 > 0$, there exists a unique solution to program (3) with sharing rule (5). This solution is characterized by two critical interest rates $\underline{r}(K_1, K_2)$ and $\bar{r}(K_1, K_2)$, and a function $p^c(K_1, K_2, r)$ such that:*

- (a) *If $r \leq \underline{r}$ then $p^c = P(x^m)$.*
- (b) *If $r \geq \bar{r}$ then no sustainable price exists that yields profits above what would be earned in the static Nash equilibrium. That is, either $\{p^c\} = \Omega = \{P(K_1 + K_2)\}$, or Ω is empty.*

¹⁸ Two papers that focus on the sharing rule and its effects on collusion are Osborne and Pitchik (1987) and Brander and Harris (1984). Both models are static and do not provide an explanation of how collusion is supported.

¹⁹ The reader is reminded that in regions c and d of Theorem 4 it may be possible to support higher prices by using a more severe punishment regime (see footnote 16). This could lead to different equilibria in the two-stage game.

(c) If $r \in (\underline{r}, \bar{r})$ then $p^c(K_1, K_2, r) = \sup \{p: Z_1(p) \leq 0\}$. Furthermore, p^c is decreasing in r .

PROOF. There are two cases to be considered. Suppose first that $K_1 + K_2 \leq x^m$. Then $\{p^c\} = \Omega = \{P(K_1 + K_2)\}$, and $\pi_i^c = \pi_i^N$. Thus, $\underline{r} = \bar{r} = 0$.²⁰

Next, turn to the case in which $K_1 + K_2 > x^m$. Define

$$\underline{r} = \sup \{r: Z_i(P(x^m)) \leq 0, \text{ for } i = 1, 2\}; \text{ and}$$

$$\bar{r} = \inf \{r: \nexists p \text{ satisfying } Z_i(p) \leq 0 \text{ and } \pi_i^c(p) > \pi_i^N, i = 1, 2\}.$$

To prove that \underline{r} and \bar{r} exist, observe that as $r \rightarrow 0$, $Z_i(P(x^m)) < 0$ for both i , and as $r \rightarrow \infty$, $Z_i(p) > 0$ for all p satisfying $\pi_i^c(p) > \pi_i^N$. The continuity of $Z_i(p)$ therefore guarantees the existence of \underline{r} and \bar{r} . Since Z_i is monotone increasing in r , it immediately follows that $P(x^m)$ is sustainable for all $r \leq \underline{r}$. For $r \geq \bar{r}$, players revert to the static Nash equilibrium. That is, either $\{p^c\} = \Omega = \{P(K_1 + K_2)\}$ (this happens when the static Nash equilibrium occurs in pure strategies), or Ω is empty (this happens when the static Nash equilibrium involves randomization).

Finally, consider part (c) of the theorem. For $r \in (\underline{r}, \bar{r})$ define p^c , the maximum sustainable price, as $p^c(K_1, K_2, r) = \sup \Omega$. From the definition of \bar{r} , we see that $\Omega \neq \emptyset$, and from the definition of \underline{r} we know $p^c < p^m$. Furthermore, since $Z_2(p) \leq 0$ for all p such that $Z_1(p) \leq 0$, the definition of p^c coincides with the one given above. One also easily shows that $Z_1(p)$ is convex for all p such that $Z_1(p) \leq 0$. Thus, p^c satisfies $Z_1(p) = 0$, and $(dZ_1/dp)(p^c) > 0$. The implicit function theorem then implies $\partial p^c / \partial r < 0$. □

The forces behind Theorem 3 are easy to understand. When the interest rate is low, firms do not discount the future heavily, and the threat of retaliation is sufficient to keep them from chiselling on the agreement, even when charging the monopoly price. As the interest rate rises, firms place less importance on future profits, and it therefore becomes more difficult to support any semi-collusive agreement. To keep firms from cheating as r rises, the cartel must lower its price so that the temptation to chisel is reduced.

Theorem 3 and equations (5) and (6) can be used to obtain the equilibrium profits in the price subgame (as a function of K_1 and K_2). These reduced form payoff functions then allow us to compute the subgame perfect Nash equilibria of the two-stage game. Unfortunately, the equation $Z_1(p) = 0$ is highly nonlinear, making it difficult to obtain analytical expressions for the maximum sustainable price and the critical interest rates. For the remainder of the paper, we therefore confine attention to the tractable case of linear demand, which can be written as:

$$(8) \quad D(p) = 1 - p$$

after a suitable choice of units. At the end of Section 5, we argue that most of our results should generalize to other demand systems satisfying our earlier assumptions.

²⁰ In the borderline case where $K_1 + K_2 = x^m$, we may also define $\underline{r} = \bar{r} = \infty$.

Theorem 5 in the Appendix reports the critical interest rates $\underline{r}(K_1, K_2)$, $\bar{r}(K_1, K_2)$ and the maximal sustainable price $p^c(K_1, K_2, r)$ for the linear demand case. One feature worth noting is that, in this case, the level of collusion that can be sustained is a decreasing function of the difference between K_2 and K_1 . In other words, as the industry becomes more symmetric, higher prices can be supported. Intuitively, as K_2 rises and K_1 falls, firm 1's share of industry profits decreases, making collusion less attractive (and cheating more attractive) to this firm. The collusive price must therefore fall in order to keep the small firm from cheating.

5. EQUILIBRIUM IN THE TWO-STAGE GAME

How much capacity should firm i purchase at time zero when it expects firm j to install capacity K_j ? To answer this question, observe that for any value of K_j its profits are given by:

$$V_i(K_i | K_j) = \pi_i^c(K_1, K_2) - cK_i = p^c \min(S_i(1 - p^c), K_i) - cK_i$$

where p^c is as in Theorem 5. Let $\bar{K}_i(K_j)$ denote the value of K_i that maximizes $V_i(K_i | K_j)$. $\bar{K}_i(K_j)$ is called firm i 's reaction function or best reply function. Firm j 's reaction function is obtained in a similar way. (K_1^*, K_2^*) then constitute a Nash equilibrium in capacities if and only if $\bar{K}_i(K_j^*) = K_i^*$.

A typical reaction function for firm 2, when the cost of capacity and the interest rate are low, is shown in Figure 1. The reaction function lies completely above the 45° line except at the point (1, 1). Thus, (1, 1) is the unique equilibrium for this case. From Theorems 3 and 5 we know that when the interest rate is low (i.e., $r < 1$) both firms will charge the monopoly price.²¹

The reaction function in Figure 1 has an unusual shape; its slope changes sign at point a and it includes concave, convex and linear segments. These properties are most pronounced when the cost of capacity and the interest rate are low and thus, we will focus mostly on this case. When capacity is cheap a firm ignores, for the most part, the cost of capacity. It thus chooses a capacity level that will support collusion and provide it with a large share of industry profits. Let us examine firm two's best reply to a value such as K_1 in Figure 1. Since r is low the monopoly price will be sustainable at $K_2 = K_1$. As K_2 rises above K_1 firm two's profits improve (because its share of cartel profits increases). At the same time, however, firm one's share falls, thereby making that firm more inclined to cheat. Eventually firm one becomes indifferent between cheating and remaining at the collusive point. Further increases in K_2 must then be accompanied by a reduction in the collusive price. From this point on, increases in K_2 increase firm two's profits because its share of cartel profits rises yet, at the same time, lower its profits due to the fall in the maximum sustainable price. The optimal value of K_2 is the value that just balances these two countervailing forces. That value is denoted by $K_2^*(K_1)$.

Suppose now that K_1 increases to K_1' . Clearly, at the point $(K_1', K_2^*(K_1'))$ higher prices are sustainable. (Since firm one's share of semi-collusive profits rises, its

²¹ Since capacity costs are effectively sunk, this monopoly price is computed as if capacity were free.

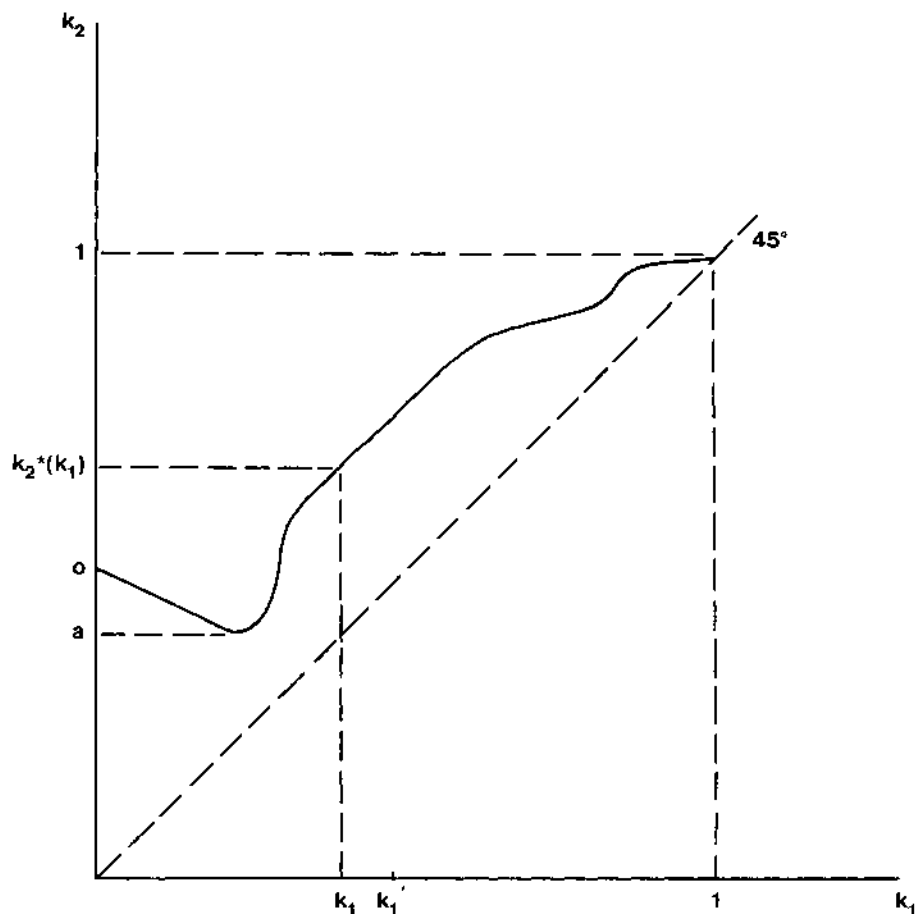


FIGURE 1

FIRM TWO'S BEST RESPONSE FUNCTION (FOR LOW VALUES OF THE COST OF CAPACITY AND THE INTEREST RATE).

temptation to cheat declines. Thus, the cartel price can increase without inducing firm one to cheat.) This implies that the optimal value of K_2 rises along with K_1 and explains why the reaction function is upward sloping beyond point a . It also provides an explanation for why the reaction function lies above the 45° line at all points except $(1, 1)$ (at $(1, 1)$ firm two cannot increase its share of industry profits by increasing capacity. Moreover $(1, 1)$ is the only symmetric point with that property).

In region oa the reaction function is downward sloping. In this region K_1 is small and thus K_2 must be small if collusion is to be sustained (otherwise firm one's share of industry profits is so small that it cheats). Theorem 5, case (b), shows that when K_1 and K_2 are both small the static Nash and semi-collusive equilibria coincide (i.e., $K_1 + K_2 \leq x^m$). Thus, each firm sells at capacity and charges the market clearing price. Gross profits are then given by $\pi_2^c(K_1, K_2) = K_2(1 - K_1 - K_2)$,

which reaches a maximum at $K_2 = (1/2)(1 - K_1)$. When the cost of capacity is small enough to be ignored, this is also the profit maximizing value of K_2 . As K_1 increases $(1/2)(1 - K_1)$ falls, explaining the negative slope in region oa . This property is thus inherited from the static Cournot-Nash reaction functions.

Increases in the interest rate or the cost of capacity affect firm two's reaction function in a similar manner. An increase in the interest rate reduces the capitalized value of the losses due to retaliation, and thus makes it more difficult to support collusion. The industry must then become more symmetric if collusive prices are to continue to characterize equilibrium. Thus, the optimal value of K_2 falls as r rises (except in region oa). An increase in the cost of capacity also causes the reaction function to shift down because it becomes more costly for a firm to increase its share of total profits. In addition, increases in r or c cause the sign reversal of the slope of the reaction function to occur later (point a moves down and to the right).

Figure 1 exhibits a unique semi-collusive equilibrium in which both firms charge the monopoly price. However, because increases in c and/or r cause the reaction function to shift down toward the 45° line, additional semi-collusive equilibria will be created when these parameter values become sufficiently high. For any given $r(c)$ there exists a critical value of $c(r)$ denoted $\hat{c}(\hat{r})$ such that if $c > \hat{c}(r > \hat{r})$ the linear segment of the reaction function— (bc) —coincides with the 45° line, creating a continuum of additional semi-collusive equilibria. In this type of equilibrium, firms charge a price above the static Bertrand-Nash level but below the monopoly price. Thus, we refer to it as a *constrained semi-collusive equilibrium* (CSE). All equilibria in which firms charge the monopoly price will be called *unconstrained semi-collusive equilibria* (USE). Once $c(r)$ reaches $\hat{c}(\hat{r})$, further increases in $c(r)$ expand the set of CSE by shifting additional portions of the reaction function down to the 45° line. Reaction functions for three different values of c are depicted in Figure 2. Lower reaction functions correspond to higher costs of capacity.

Referring to the same figure, a third type of equilibrium emerges when c and/or r rise even further. This type appears whenever the reaction function's downward sloping portion reaches the 45° line before its slope changes sign. For any given $r(c)$ there thus exists a critical value of $c(r)$ denoted $\hat{\hat{c}}(\hat{\hat{r}})$ such that if $c > \hat{\hat{c}}(r > \hat{\hat{r}})$ equilibria of this type will be present. In these equilibria no price above the static Bertrand-Nash level is supportable. We will therefore refer to them as *noncollusive equilibria* (NCE). For any given value of r and c there is at most one NCE.

To summarize, three types of semi-collusive equilibria may occur: unconstrained semi-collusive equilibria in which the monopoly price is charged, constrained semi-collusive equilibria in which a price above the static Bertrand-Nash level but below the monopoly level is charged, and noncollusive equilibria in which no price above the static Bertrand-Nash level is supportable. Equilibria of the USE type are characterized by high levels of capacity and excess capacity while equilibria of the NCE variety are characterized by low levels of capacity and no excess capacity. In the USE case the excess capacity provides firms with a powerful weapon for retaliation and thus facilitates collusion. There is at most one USE and at most one NCE, but neither need exist.

Constrained semi-collusive equilibria are characterized by smaller capacity and excess capacity levels than USE. In addition, there is a continuum of CSE

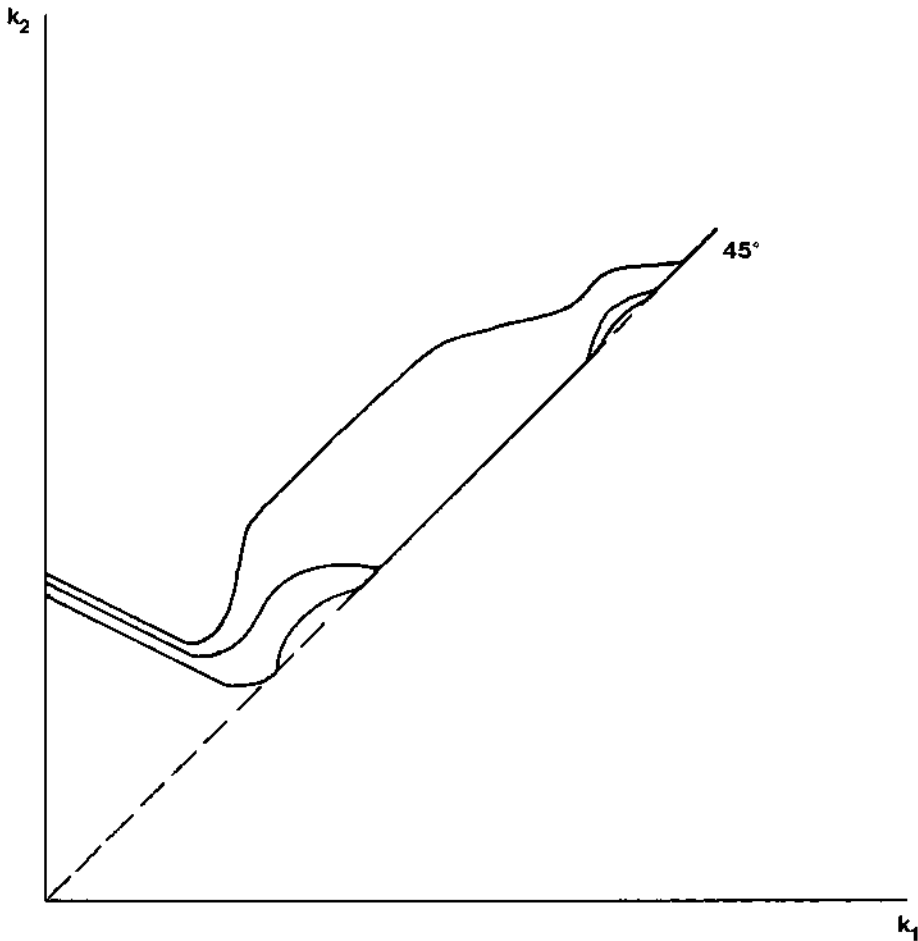


FIGURE 2

FIRM TWO'S BEST RESPONSE FUNCTION FOR VARIOUS VALUES OF THE COST OF CAPACITY c .

whenever they are present. Raising r or c creates more equilibria of the CSE variety when the level of c is not too high. However, for large values of c increases in c and r tend to reduce the number of CSE.

In Figure 3 the values of c and r which are consistent with equilibria of each type are shown. It is clear from the figure that more than one type of semi-collusive equilibrium may be present at any one time. On the other hand, the semi-collusive equilibria can always be Pareto ranked. If we restrict our attention to the set of undominated equilibria, we are left with unique capacity levels for most values of the parameters. Figure 4 illustrates, for given values of c and r , the price charged in the equilibrium that is not Pareto dominated by any other semi-collusive equilibrium. As is evident from Figure 3, increases in c or r may, *ceteris paribus*, reduce the level of collusion and hence of equilibrium prices. Such reductions are

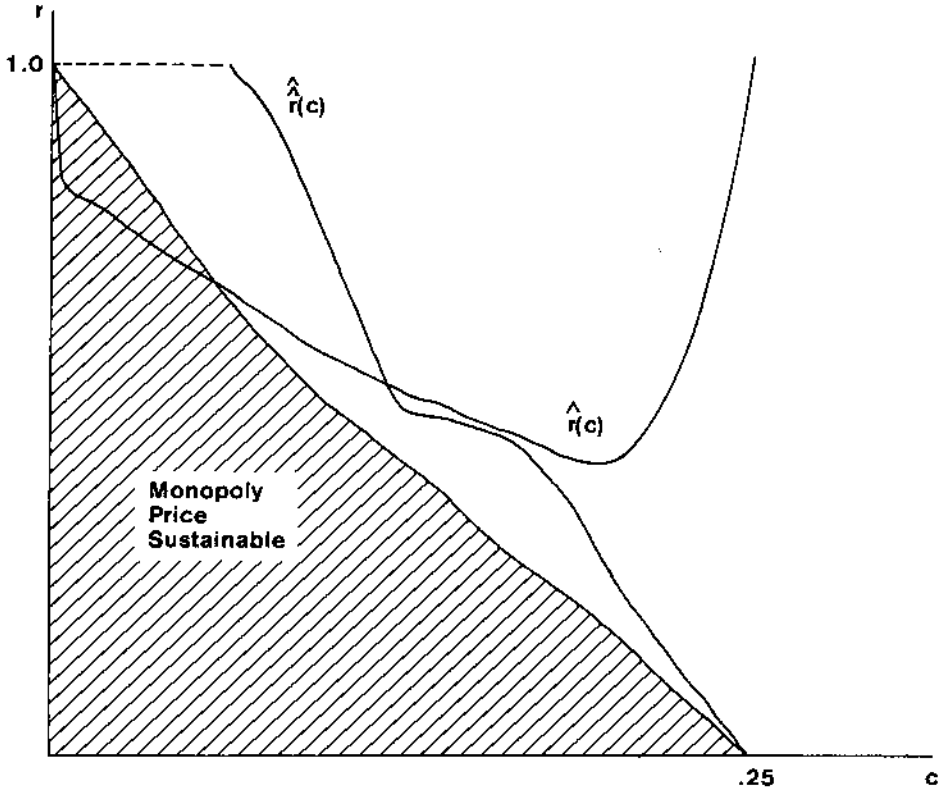


FIGURE 3

REGIONS OF EXISTENCE OF USE (MONOPOLY PRICE SUSTAINABLE), CSE ($r < \hat{p}$) AND NCE ($r > \hat{p}$), AS A FUNCTION OF THE COST OF CAPACITY c AND THE INTEREST RATE r .

always accompanied by a fall in the levels of capacity and excess capacity. In other words, lower levels of excess capacity coincide with *lower* levels of collusion. This positive correlation was discussed at length in Section 2.

At this point, it is instructive to reflect on what properties (if any) of the linear demand example carry over to a more general setting. Benoit and Krishna's result (1987, Lemma 6) implies that in any symmetric equilibrium of the two-stage game in which firms do not implement the static Cournot-Nash equilibrium, there is excess capacity. Moreover, it is clear that the comparative static properties of our model (the relationship between the degree of collusion and the amount of excess capacity held by an industry) will generalize. On the other hand, we have been unable to generalize the result that all semi-collusive equilibria must involve capacity choices no lower than the static Cournot-Nash level (taking into account the cost of capacity). What remains true, however, is that joint capacities must exceed the monopolistic level.

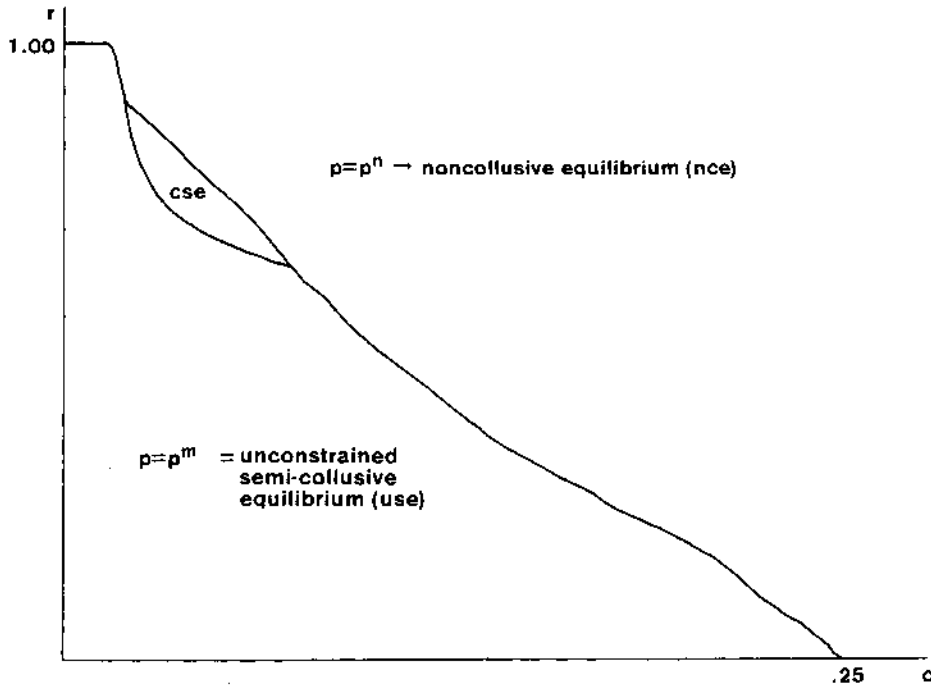


FIGURE 4

NATURE OF THE PARETO-OPTIMAL COLLUSIVE EQUILIBRIUM, AS A FUNCTION OF THE COST OF CAPACITY c AND THE INTEREST RATE r .

6. CONCLUSION

In this paper we analyzed a model of oligopolistic competition in which firms' capital decisions play a crucial role in determining the level of collusion that can be supported in equilibrium. As emphasized by Benoit and Krishna (1987), when capacities are subject to substantial cost of adjustment while prices are not, firms generally carry excess capacity in equilibrium. If the interest rate is low and/or capacity is cheap, producers choose a scale of operation large enough to support a monopoly price that ignores capacity costs. As the interest rate or the cost of capacity rise, it becomes too costly to carry enough excess capacity to support this price. Equilibrium capacities and the level of collusion thus fall. Eventually, it becomes impossible to support any collusion at all, and we recover the (static) Cournot-Nash equilibrium.

While low values of the interest rate allow tight price collusion in our model, equilibrium profit levels need not be close to their fully cooperative equivalents. There are two reasons for this. First, as emphasized by Benoit and Krishna, firms need excess capacity in order to support collusive outcomes. This extra cost of sustaining a given profit stream would not have to be incurred by a monopolist. Second, some rents are dissipated as firms expand capacities in order to increase their share of collusive profits. This capacity "race" is aggravated in our model

because we do not allow firms to punish excessive capacity choices with subsequent price wars. While firms could, in principle, avoid this kind of miscoordination, empirical evidence suggests they have trouble doing so. But even when tacit cooperation and monitoring in all dimensions is feasible, the capacity race is still on. For, as shown in Benoit and Krishna, any implicitly collusive agreement must lead to aggregate capacities exceeding the level that a monopolist would choose.

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APPENDIX

In order to calculate $\bar{r}(K_1, K_2)$, $\bar{p}(K_1, K_2)$ and $p^c(K_1, K_2, r)$ we first need analytic expressions for π_i^N , the static Bertrand-Nash equilibrium profits. For the linear demand case, these profit functions were derived in Kreps and Scheinkman (1983) and Osborne and Pitchik (1986). Their results are collected in Theorem 4 below (note that when the equilibrium occurs in mixed strategies, the equilibrium distribution functions are not given, since they are not needed here. The interested reader is referred to any of the above papers.)

THEOREM 4. *For each pair (K_1, K_2) with $K_1 \leq K_2$, the static price-setting game with capacity constraints has a unique static Nash equilibrium:*

- (a) *If $K_1 \geq 1$ the equilibrium is in pure strategies, both firms charge $p = 0$ and earn zero profits.*
- (b) *If $K_2 \leq \frac{1}{2}(1 - K_1)$ the equilibrium is in pure strategies, both firms charge $p = 1 - K_1 - K_2$ and profits are given by $\pi_i^N = K_i(1 - K_1 - K_2)$ for $i = 1, 2$.*
- (c) *If $\frac{1}{2}(1 - K_1) \leq K_2 \leq \frac{1}{2}[1 + \sqrt{K_1(2 - K_1)}]$ the equilibrium is in mixed strategies, and profits are given by $\pi_i^N = (K_i/4K_2)(1 - K_1)^2$.*
- (d) *If $K_2 \geq \frac{1}{2}[1 + \sqrt{K_1(2 - K_1)}]$ and $K_1 \leq 1$ the equilibrium is in mixed strategies, profits are given by $\pi_2^N = \frac{1}{4}(1 - K_1)^2$ and $\pi_1^N = (K_1/2)[1 - \sqrt{K_1(2 - K_1)}]$.*

Theorem 4 and equations (5), (6) and (7) provide the information to calculate the critical interest rates and the maximum sustainable price. We do this in Theorem 5, but omit the computations for the sake of brevity. To facilitate the statement of this theorem, we introduce the following notation:

$$\alpha = 1 - \sqrt{K_1(2 - K_1)}, \quad z = 2(K_1 + \min(K_2, 1)) - 1,$$

$$\theta = \frac{1}{1+r} [1 + r - r(K_1 + K_2)], \quad \phi = 2[1 + r - r(K_1 + \min(K_2, 1))],$$

$$\delta = \frac{1}{K_2(1+r)} (1 - K_1)^2(K_1 + K_2), \quad \psi = 8\alpha(1+r)(K_1 + K_2).$$

THEOREM 5. For each triple (K_1, K_2, r) with $K_1 \leq K_2$ and for every $F(\cdot, \cdot)$ satisfying the property $F_1, F_2 > 0$ there is a unique solution to program (3) with sharing rule (5), given by:

$$p = \begin{cases} \frac{1}{2} & \text{if } r \leq \underline{r} \\ p^c(K_1, K_2, r) & \text{if } \underline{r} \leq r \leq \bar{r}, \end{cases}$$

and when $r > \bar{r}$ the equilibrium is as in Theorem 4. Moreover:

(a) If $K_1 \geq 1$, then $\underline{r} = \bar{r} = 1$.

(b) If $K_2 \leq \frac{1}{2}(1 - K_1)$, then
if $K_2 \geq \frac{1}{2} - K_1$,

$$\underline{r} = z; \quad \bar{r} = \frac{z}{1 - K_1 - K_2}; \quad p^c(K_1, K_2, r) = \frac{K_1 + K_2}{1 + r};$$

if $K_2 \leq \frac{1}{2} - K_1$, the static Nash equilibrium price is equal to the monopoly price (that is, $\bar{r} = \underline{r} = 0$).

(c) If $\frac{1}{2}(1 - K_1) \leq K_2 \leq \frac{1}{2}[1 + \sqrt{K_1(2 - K_1)}]$ then

$$\underline{r} = \begin{cases} \left(\frac{K_1}{K_2}\right)^2 [K_2(2 - K_1) - (1 - K_1)^2] & \text{if } K_1 \geq \frac{1}{2} \\ \frac{1}{K_2 z} [K_2 - (1 - K_1)^2(K_1 + K_2)] & \text{if } K_1 \leq \frac{1}{2} \end{cases},$$

$$\bar{r} = \frac{K_1}{K_2}, \quad p^c(K_1, K_2, r) = \frac{1}{2} \{\theta + \sqrt{\theta^2 - \delta}\}.$$

(d) If $K_2 \geq \frac{1}{2}[1 + \sqrt{K_1(2 - K_1)}]$ and $K_1 \leq 1$ then

$$\bar{r} = \begin{cases} \max \{0, ((z + 1)(1 - \alpha)/z) - 1\} & \text{if } K_1 \leq \frac{1}{2} \\ \max \{0, K_1(1 - \alpha(1 + z))/\min(K_2, 1)\} & \text{if } K_1 \geq \frac{1}{2} \end{cases},$$

$$\bar{r} = \max \left\{ 0, \frac{1 - \alpha - [\alpha^2 + 2\alpha(K_1 + \min(K_2, 1) - 1)]^{1/2}}{K_1 + \min(K_2, 1) - (1 - \alpha) + [\alpha^2 + 2\alpha(K_1 + \min(K_2, 1) - 1)]^{1/2}} \right\},$$

$$p^c(K_1, K_2, r) = \frac{1}{4(1 + r)} [\phi + \sqrt{\phi^2 - \psi}].$$

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