The Structure of Simple General Equilibrium Models with Frictional Unemployment

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We develop a two-sector general equilibrium model in which equilibrium unemployment arises endogenously because of trading frictions in the labor market of one sector. Externalities inherent in the search process lead to inefficient equilibria, and this has important implications for the basic structure of the economy. In particular, the relationship between factor rewards and commodity prices is fundamentally different from the analogous relationship in a frictionless economy. One implication is that the economy's relative supply curve may be downward sloping, especially when the search sector is small. We also present several applications of the analysis.

I. Introduction

Because of its simple structure and intuitive appeal, the two-sector general equilibrium Walrasian model has served "as the workhorse for most of the developments in the pure theory of international trade" (Jones 1965, p. 557). This model has also been used extensively in most of the other applied fields of economics. In his seminal article "The Structure of Simple General Equilibrium Models," Jones contributed to our understanding of this model by exposing its basic structure in a manner that allowed him to unify the many approaches that had been developed in different applied areas. In particular, he

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examined two key relationships that govern the behavior of the supply side of a competitive economy. The first relationship links factor endowments, factor rewards, and output levels and is derived from the requirement that factor markets clear. The second relationship is between factor rewards and commodity prices and results from the fact that unit production costs must equal output prices in the presence of constant returns to scale. By carefully examining these relationships, it is possible to derive several of the fundamental theorems of international economics (e.g., the Rybczynski and Stolper-Samuelson theorems). In addition, the effects of parametric changes in the economy (e.g., tax rates) can be easily understood by examining how such changes work through these two basic relationships.

While the Walrasian model is exceedingly useful for many types of analysis, one may be reluctant to rely on it to analyze situations in which unemployment is a consideration: factors of production are always fully employed in the full-information, frictionless markets. Since casual observation suggests that the effects of a policy on the unemployment rate and the welfare of the unemployed are often major considerations for policymakers, it is vitally important to augment the Walrasian approach in a way that permits the investigation of these issues. The purpose of this paper is to do just that. In particular, we present and analyze a simple two-sector general equilibrium model that allows for equilibrium unemployment. Our main goal is to understand how our model differs in structure from the model analyzed by Jones (which we refer to as the Jones model).[1] Our analysis indicates that introducing frictional unemployment creates the potential for the structural relationships within the economy to be qualitatively different from the analogous relationships in a frictionless economy.

We model unemployment by assuming that in one sector, factor markets are frictionless so that the duration of unemployment is zero, while in the other sector, idle factors of production must search each other out in order to produce.[2] Factors are mobile across sectors and

[1] More accurately, we compare our model with the standard two-sector, two-factor model using an approach that closely resembles the approach adopted by Jones in his masterful exposition and synthesis of earlier work. Our reference to the "Jones model" is not intended to imply that Jones was the first to formalize, develop, or use the model.

[2] This approach was originally suggested by Friedman (1968) in his classic paper "The Role of Monetary Policy," in which he defined the natural rate of unemployment as "the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the costs of mobility, and so on." This definition has recently been made operational by Diamond in a series of papers (1981, 1982, 1984*a*, 1984*b*) in which he models unemployment as the result of problems in coordinating exchange. He accom-

in equilibrium distribute themselves so that the expected lifetime return is the same in both sectors. The trading frictions in the search sector represent the *only* difference between our model and the Jones model, and yet we demonstrate that in some cases this leads to an economy with a surprisingly different structure. In particular, we show that the relationship between factor rewards and commodity prices is fundamentally different in an economy with search-generated unemployment. This leads to the possibility of downward-sloping relative supply curves, and, in fact, we demonstrate that such a phenomenon is likely to occur if the search sector is relatively *small.* If, however, the search sector is large enough, the structure of the economy will be qualitatively the same as the structure of the frictionless general equilibrium model.

In the next section, we introduce the model, define equilibrium, and briefly discuss its efficiency properties. Equilibrium is generally *not* efficient, and this is one of the driving forces behind our results. In Section III, we derive the two fundamental relationships that describe the structure of the model and compare them with their counterparts in the Jones model. In order to gain some insight into the forces behind our results, we devote Section IV to a detailed investigation of the nature of the production process in the search sector. In this section, we demonstrate that the equilibrium factor intensity employed in the search sector is not optimal. Changes in factor prices alter this factor intensity and may enhance or inhibit efficiency. When the search sector is small these efficiency effects are large enough to dominate the traditional mechanism linking commodity and factor prices. As the sector grows in size, this effect shrinks relatively and the economy eventually behaves in a manner similar to a pure Walrasian economy. Some applications of the analysis are discussed in Section V, and concluding remarks are offered in Section VI.

II. The Model

A. *Endowments, Production Technology, and Preferences*

Our economy consists of two sectors (X and Y) and two factors of production (A and B). Each factor is endowed with one (indivisible)

plishes this by assuming that it takes time and effort for potential trading partners to find each other. Our approach is therefore very similar to Diamond's. However, his purpose in developing these models was to investigate the macroeconomic properties of economies with a natural rate of frictional unemployment, and he was able to do so in a remarkably simple setting: a barter economy with one good and one factor of production. The questions that we wish to address are fundamentally different and require a somewhat more elaborate model.

unit of leisure in each period that she may either consume or offer as labor. Each worker is finitely lived, although the age of a worker at death is a random variable. The probability that any individual worker of either type dies in any given period is time invariant and equal to d, which also equals the birth rate for both types of workers. All workers are risk neutral.

We assume that the production function corresponding to good Y is twice continuously differentiable and characterized by constant returns to scale:

$$Y = Y(L_{ay}, L_{by}), \tag{1}$$

where L_{iy} is the amount of type i labor employed in the production of Y.

Finally, we assume that the production of one unit of X requires exactly one worker of each type. We refer to a pairing of opposites as a "match" and designate the number of matches as L_m, so that $X = L_m$.

B. The Search Process

Workers in sector Y are immediately hired in full-information, competitive auction markets, while workers who seek employment in sector X must search each other out. If a type i worker in sector X fails to locate a worker of the opposite type, she remains idle (unemployed) for that period. A worker in this sector engages in search at the start of every period in which she is unemployed. Matches survive as long as both workers live. On the death of either partner, the survivor again engages in search.[3]

We assume that the number of new matches created every period (L_n) is a function of L_{az} and L_{bz}, the numbers of workers of each type who begin the period searching. This function is characterized by constant returns to scale, positive marginal products, and symmetry such that $L_n(L_{az}, L_{bz}) = L_n(L_{bz}, L_{az})$.[4] In this case, the per period probability that a type i searcher finds a match can be written as $e_i(s)$, where s denotes the proportion of searchers who are of type A. Our assump-

[3] Consistent with the bulk of the literature in this area, deaths are used to capture the role of exogenous separations in factor markets (see, e.g., Diamond 1981, 1982; Mortensen 1982; Pissarides 1984).

[4] Empirical support for the assumption of constant returns to scale is provided by Nickell (1979). The implications of increasing returns to scale in the search technology are discussed at length in Diamond (1984*b*). The substance of our results does not depend on the symmetry assumption. In fact, in n. 16 we show how our results can be generalized to situations in which the matching function is not symmetric.

tions about the search technology imply that $e_a'(s) < 0 < e_b'(s)$ and that $e_i(s) = e_j(1 - s)$ for $j \neq i$.[5]

C. *Factor Returns*

Since sector Y markets are frictionless and competitive, all workers in this sector are always employed and earn the value of their marginal product. If we let w_{iy} denote the wage paid to a type i worker in this sector and P_y the price of good Y, then this condition is expressed as

$$P_y Y_i = w_{iy} \quad \text{for } i = a, b, \tag{2}$$

where Y_i is the partial derivative of Y( ) with respect to L_{iy}. Letting V_{iy} denote the expected lifetime income to a type i worker in sector Y, we have

$$V_{iy} = \frac{w_{iy}}{d} \quad \text{for } i = a, b. \tag{3}$$

We assume that the proceeds from the sale of a unit of X are divided between the two matched workers who produced it according to the Nash cooperative bargaining solution so that the surplus created by the match is evenly split.[6] For future use, we let α_i denote the share of the proceeds that go to the type i worker and P_x the price of X.

To describe the solution to the bargaining problem formally, let V_{is} denote the expected lifetime income to a type i worker currently searching in sector X and V_{im} the expected lifetime income to a type i

[5] We first note that e_i, the probability that a randomly chosen type i searcher becomes matched, is equal to L_n/L_{is}, where $L_n(L_{as}, L_{bs}) \leq \min(L_{as}, L_{bs})$. Dividing the numerator and denominator by the total number of searchers $(L_{as} + L_{bs})$ permits us to express e_i solely as a function of s, the proportion of searchers who are type A agents; i.e., $s = L_{as}/(L_{as} + L_{bs})$. Next, symmetry follows because

$$e_a(s) = \frac{L_n(s, 1 - s)}{s} = \frac{L_n(1 - s, s)}{s} = e_b(1 - s).$$

Moreover,

$$e_a'(s) = \left(\frac{1}{s^2}\right)(sL_{n1} - sL_{n2} - N) = -\frac{L_{n2}}{s^2} < 0,$$

where the numeric subscript refers to partial differentiation and the last equality follows from the assumption of constant returns to scale. Finally, we note that constant returns to scale in the matching technology implies $e_a(s)/e_b(s) = (1 - s)/s$ for all s.

[6] This assumption is consistent with a bargaining process in which the agents exchange sharing rules until one agent makes an offer that is acceptable to her partner, and the time it takes to make counteroffers is arbitrarily close to zero. See the work of Binmore (1982), McLennan (1982), Rubinstein (1982), and Rubinstein and Wolinsky (1985).

worker currently matched in sector X. Then, under the assumption that unemployed workers earn no income,[7]

$$V_{is} = e_i(s)V_{im} + [1 - e_i(s)](1 - d)V_{is}, \tag{4}$$

$$V_{im} = \alpha_i P_x + (1 - d)^2 V_{im} + d(1 - d)V_{is} \quad \text{for } i = a, b. \tag{5}$$

In (4), $[1 - e_i(s)](1 - d)$ represents the probability of failing to find a match but surviving to search again next period. In (5), $(1 - d)^2$ represents the probability that both partners survive the period so that they begin the next period still matched and $d(1 - d)$ represents the probability that the type j worker will die and the type i worker will survive (in which case the type i worker must begin searching again). Equations (4) and (5) can be solved for V_{is} and V_{im} to obtain

$$V_{is} = \frac{e_i \alpha_i P_x}{d[1 - r(1 - e_i)]}, \tag{6}$$

$$V_{im} = \frac{[1 - (1 - d)(1 - e_i)]\alpha_i P_x}{d[1 - r(1 - e_i)]} \quad \text{for } i = a, b, \tag{7}$$

where the argument of e_i has been suppressed and we have defined $r \equiv (1 - d)^2$.

We are now in a position to describe how α_i is determined. As previously indicated, the Nash bargaining solution evenly divides the surplus created by the match. The surplus is the excess of expected lifetime income if matched over the next-best alternative, namely, waiting a period and searching again. The surplus generated for a type i worker is

$$V_{im} - (1 - d)V_{is} = \frac{\alpha_i P_x}{1 - r(1 - e_i)} \quad \text{for } i = a, b. \tag{8}$$

Equating $V_{am} - (1 - d)V_{as}$ with $V_{bm} - (1 - d)V_{bs}$ and solving for α_a, we obtain

$$\alpha_a = \frac{1 - r(1 - e_a)}{2 - r(2 - e_a - e_b)}. \tag{9}$$

When output prices and the mix of searchers are given, this value of α_a solves the Nash cooperative bargaining problem. Of course, $\alpha_b = 1 - \alpha_a$.

D. *Equilibrium*

In any steady-state equilibrium, the number of type i workers who enter sector k through birth must equal the number who exit the

[7] This assumption is made to keep the algebra as simple as possible; it is not essential for any of our results.

sector because of death. In addition, the number of searchers of each type and the number of matches must be time invariant. If we let β_i denote the proportion of newly born type i workers who choose to seek employment in sector X, L_i the number of type i workers, and L_{is} the number of type i searchers remaining after the matching process for the period has ended, then these conditions imply

$$(L_m + L_{is})d = \beta_i L_i d, \tag{10}$$

$$L_m = rL_m + d(1 - d)e_i L_m + \beta_i L_i e_i d + L_{is}(1 - d)e_i. \tag{11}$$

In (10), $L_i d$ is the number of newly born type i workers and $(L_m + L_{is})d$ is the number of type i sector X workers who die each period. If this condition holds, the sector sizes remain constant over time. In (11), rL_m is the number of matches that survive from the previous period; $\beta_i L_i e_i d$ is the number of new entrants who immediately find partners; $d(1 - d)e_i L_m$ is the number of type i workers who survived, had partners who died, and immediately found new matches; and $L_{is}(1 - d)e_i$ is the number of type i searchers who survived and found partners. These last three terms represent all workers who begin period t as searchers but end the period matched.[8] The sum of all four terms on the right-hand side yields the number of matches at time t, which must equal the number at time $t - 1$, given on the left-hand side of (11). If this condition holds, the number of matches and the number and mixture of searchers will be time invariant.

In addition to these steady-state conditions, a nonspecialized production equilibrium for this economy is characterized by (i) zero profits in sector Y, (ii) indifference between sectors for both types of workers, and (iii) zero excess demand for labor of each type in sector Y.[9]

The zero-profit condition for sector Y is given by

$$P_y = l_{ay}w_{ay} + l_{by}w_{by}, \tag{12}$$

where l_{ik} denotes the unit input requirement of type i workers in sector k.

For a type i worker to be indifferent between sectors, the expected lifetime return to seeking employment in either sector must be the same. More exactly, since a worker cannot choose *employment* in sector

[8] In computing s, it is crucial to distinguish the number of workers who *begin* the period as searchers from L_{is}, the number who remain searching after the completion of the matching process.

[9] We should point out that, regardless of relative output prices, specialization in good Y is always a production equilibrium. This follows from the fact that if everyone in the economy except one worker seeks employment in sector Y, it will be impossible for that worker to produce in sector X. Thus that worker will take a job in sector Y regardless of the relative product prices.

X (she can choose only to search), the second equilibrium condition is given by

$$V_{iy} = V_{is} \quad \text{for } i = a, b. \tag{13}$$

The final equilibrium condition is met when labor supply equals demand:

$$L_i = L_{iy} + L_{is} + L_m \quad \text{for } i = a, b. \tag{14}$$

This completes the description of the model. We emphasize that the only difference between this and the Jones model is that search is required to find employment in sector X. If we set $e_i(s) = 1$ for both i and for all s, the model would reduce to the standard model with Leontief technology in sector X.

E. Efficiency

Diamond (1982) and others have shown that when search is required to find trading opportunities, externalities are generated that lead to inefficiencies. In the context of the model presented here, it can be demonstrated that in equilibrium the search sector is too small and its factor intensity is too asymmetric (see Davidson, Martin, and Matusz 1987). The reason for this is that whenever a match occurs, the two partners *each* enjoy an increase in expected lifetime income measured by $V_{im} - V_{is}$ for $i = a, b$. However, when a worker contemplates entering sector X, she ignores the increase in her partner's expected income that will be generated by every partnership that she enters. Workers therefore ignore a positive externality associated with entering this sector so that it is too small in equilibrium. Moreover, the externalities are not of equal magnitude. If the majority of searchers are type i workers, then, in general, the type j workers ignore a larger external effect than their type i counterparts. Sector X then becomes too i-intensive. Making sector X more j-intensive moves the economy toward the production possibilities frontier as the economy utilizes the search technology more efficiently. As we demonstrate in Section IV, it is this externality that, when sector X is relatively small, leads the economy to behave in a fundamentally different manner than a frictionless economy.

III. Hat Calculus

In this section, we derive the equations of change that relate the prices of factors and goods to output quantities.[10] For convenience, we in-

[10] Our comparative static exercises are actually comparisons across steady states. When evaluating the effect of a change in a parameter, we ignore the period of convergence to the new steady state. To take into account this period of convergence, the analysis would have to be modified along the lines of Diamond (1980).

troduce the following additional notation: $L_{ix} = L_{in} + L_{is}$ = the total number of type i workers in sector X; $l_k = L_{ak}/L_{bk}$ = the A-intensity of sector k; θ_{ik} = the value share of factor i in industry k (e.g., $\theta_{ay} = w_{ay}l_{ay}/P_y$); $\theta = \theta_{ax} - \theta_{ay} = \theta_{by} - \theta_{bx} = \alpha_a - \theta_{ay}$; $\lambda_{ik} = L_{ik}/L_i$ = the physical share of factor i in industry k; $\lambda = \lambda_{ax} - \lambda_{bx} = \lambda_{by} - \lambda_{ay}$; σ_k = the elasticity of substitution in sector k; $\phi_i = e_i's/e_i$ = the elasticity of e_i; and $\phi = \phi_a - \phi_b < 0$.

We begin with a brief review of the key equations of the Jones model and an explanation of how these relationships are altered by the introduction of trading frictions. There are six key equations in the Jones model: two factor-market-clearing conditions, two zero-profit conditions, and two cost minimization conditions.[11] In the present context, sector Y is modeled in the standard way, consisting of perfectly competitive profit-maximizing firms with constant returns to scale technology. In addition, factor markets in sector Y are frictionless. Therefore, the zero-profit and cost minimization conditions for this sector are unchanged (eqq. [12] and [2], respectively). The factor-market-clearing conditions (eqq. [14]) are also unchanged *provided* that factor usage in sector X is taken to include both matched and unmatched factors. Any differences in the models must therefore be the result of differences in the sector X zero-profit and cost minimization conditions.

The analogue of the sector X zero-profit condition can be derived from (13), the equation that states that workers must be indifferent between sectors in equilibrium. We obtain (see result A1 of the Appendix)

$$P_x = l_{ax}w_{ay} + l_{bx}w_{by}. \tag{15}$$

By examining (13) in some detail, we can provide an intuitive interpretation of (15).[12] In the Appendix we demonstrate that (13) may be written as $\alpha_i P_x = l_{ix}w_{iy}$, where $l_{ix} = (L_{is} + L_m)/L_m$. Define $\pi_{ix} \equiv 1/l_{ix}$ so that π_{ix} represents the probability that a sector X, type i factor will be matched at the *end* of the period (note that $\pi_{ix} \neq e_i(s)$ since the latter represents the probability that a factor, initially unmatched, will find a match during the period). Next, define $w_{ix} \equiv \alpha_i P_x$ so that w_{ix} is the wage earned in sector X by a type i factor that is matched. Then (13) states that, in equilibrium, arbitrage by factors across sectors requires the certain return in sector Y to equal the (unconditional) expected return in sector X: $w_{iy} = \pi_{ix}w_{ix} + (1 - \pi_{ix}) \cdot 0$. Equation (15) is obtained by solving for w_{ix} and summing w_{ax} and w_{bx}.

[11] Jones actually begins his analysis at a more fundamental level, explicitly pointing out the dependence of the input requirements on factor prices. The six equations that we describe can be derived from his slightly larger system.

[12] We would like to thank an anonymous referee for this interpretation.

Finally, we turn to the analogue of the cost minimization condition for sector X. In the standard model, firms choose their mix of inputs to minimize unit production cost. In our model, however, the factor intensity in the search sector is governed not by a cost minimization condition, but by the steady-state conditions (10) and (11). From these equations we obtain

$$l_{ix} = \frac{1 - r(1 - e_i)}{e_i}. \quad (16)$$

In Section II we noted that the equilibrium factor intensities in our model are not optimal. Therefore, the value of l_{ix} in (16) is not the value that minimizes the cost of producing a given amount of X. As we will see below, this is the driving force behind our results.

We are now in a position to derive the equations of change. We first develop the relationship between factor rewards and output quantities by rewriting (14), the factor-market-clearing condition, as

$$L_i = l_{ix}\mathrm{X} + l_{iy}\mathrm{Y}. \quad (17)$$

Logarithmic differentiation of this condition yields

$$\hat{L}_i = \lambda_{ix}\hat{\mathrm{X}} + \lambda_{iy}\hat{\mathrm{Y}} + \lambda_{ix}\hat{l}_{ix} + \lambda_{iy}\hat{l}_{iy}, \quad (18)$$

where the circumflex denotes the proportionate change in a variable.

We use the equilibrium conditions to express $\hat{l}_{ik}$ as a function of the changes in sector Y wages. We begin by differentiating (16) to obtain

$$\hat{l}_{ix} = -\frac{\phi_i(1 - r)\hat{s}}{1 - r(1 - e_i)}. \quad (19)$$

Next, we can use equations (3), (6), (9), and (13) to solve for the equilibrium wages as a function of s. Doing so, we obtain $w_{ay}/w_{by} = e_a/e_b$.[13] Differentiation yields

$$\hat{s} = \frac{\hat{w}_{ay} - \hat{w}_{by}}{\phi}. \quad (20)$$

Substitution of (20) into (19) yields $\hat{l}_{ix}$ as a function of $\hat{w}_{ay} - \hat{w}_{by}$.

[13] The relationship states that relative sector Y wages equal relative ex ante employment probabilities for those initially unemployed. The latter ratio depends on the relative proportions of factors among the unemployed. This dependence makes sense in that unemployment in sector X is the relevant alternative for employed sector Y workers. On the other hand, relative sector X wages (for matched workers) equal the ratio of unconditional probabilities of employment: $w_{ax}/w_{bx} = \pi_{ax}/\pi_{bx} = l_x$. In this case, the latter ratio is the sectoral factor intensity, including the employed *and* the unemployed. The reason for this is that sector X wages are the result of a bargaining process. In this process, the factor that is more abundant in the sector negotiates from a weaker position because if the match were to dissolve, she would have a more difficult time finding a new partner.

Similarly, cost minimization in sector Y implies

$$\hat{l}_{iy} = -\theta_{jy}\sigma_y(\hat{w}_{iy} - \hat{w}_{jy}). \quad (21)$$

Finally, substituting (19)–(21) into (18) and setting $\hat{L}_i = 0$ yields

$$\lambda_{ax}\hat{X} + \lambda_{ay}\hat{Y} = (\hat{w}_{ay} - \hat{w}_{by})(q_a + \lambda_{ay}\theta_{by}\sigma_y), \quad (22)$$

$$\lambda_{bx}\hat{X} + \lambda_{by}\hat{Y} = (\hat{w}_{ay} - \hat{w}_{by})(q_b - \lambda_{by}\theta_{ay}\sigma_y), \quad (23)$$

where $q_i = \lambda_{ix}\phi_i(1 - r)/\phi[1 - r(1 - e_i)]$.

The relationship between factor rewards and output quantities is now found by subtracting (23) from (22). The resulting expression is

$$\lambda(\hat{X} - \hat{Y}) = (\hat{w}_{ay} - \hat{w}_{by})(q_a - q_b + t_y\sigma_y), \quad (24)$$

where $t_k = \theta_{bk}\lambda_{ak} + \theta_{ak}\lambda_{bk} > 0$.

Compare (24) with the result implied by equations (1b) and (2b) of Jones (1965), reported here (with obvious changes of notation) as

$$\lambda(\hat{X} - \hat{Y}) = (\hat{w}_a - \hat{w}_b)(t_x\sigma_x + t_y\sigma_y). \quad (25)$$

Because there are no factor market frictions in the Jones analysis, w_i denotes the payment to factor i regardless of sector.

By inspection, the only difference between (24) and (25) is that $q_a - q_b$ replaces $t_x\sigma_x$. However, $q_a > 0$ and $q_b < 0$. Therefore, $q_a - q_b > 0$, and it follows that the mechanism linking output levels to factor rewards in our model is *qualitatively identical* to the mechanism at work in the standard two-sector general equilibrium model. In both cases, the qualitative effect of a change in relative outputs on relative factor returns depends on the relative *physical* factor intensities of the sectors (i.e., the sign of λ).

We now turn to the derivation of the equation that relates commodity and factor prices in our model. Totally differentiating (15), the sector X pricing equation, and using (19) and (20), we obtain

$$\hat{P}_x = \theta_{ax}\hat{w}_{ay} + \theta_{bx}\hat{w}_{by} - (\hat{w}_{ay} - \hat{w}_{by})\Sigma, \quad (26)$$

where $\theta_{ix} = \alpha_i$ and $\Sigma = (\phi_a + \phi_b)(1 - r)/\phi[2 - r(2 - e_a - e_b)]$.

Similarly, we differentiate (12) to obtain an expression for $\hat{P}_y$:

$$\hat{P}_y = \theta_{ay}\hat{w}_{ay} + \theta_{by}\hat{w}_{by}. \quad (27)$$

Finally, subtracting (27) from (26) provides the link between factor prices and product prices when factor markets exhibit frictional unemployment:

$$\hat{P}_x - \hat{P}_y = (\theta - \Sigma)(\hat{w}_{ay} - \hat{w}_{by}). \quad (28)$$

Now, from (3b) and (4b) of Jones (1965), we can derive

$$\hat{P}_x - \hat{P}_y = \theta(\hat{w}_a - \hat{w}_b). \quad (29)$$

Comparing (28) with (29), we see that the link between commodity and factor prices in our model is fundamentally different from that in the standard model. In the latter, the qualitative effect of a change in commodity prices on factor returns depends on the relative *value* factor intensities (i.e., the sign of θ). As we show below, the Σ term in (28) complicates this link in a manner that has significant implications for the shape of the relative supply curve (i.e., the supply-side relationship between X/Y and P_x/P_y).

In a frictionless, nondistorted world, increases in the relative price of X *always* cause the supply of X to rise and the supply of Y to fall. This is seen by combining (25) and (29) to yield

$$\hat{X} - \hat{Y} = (\hat{P}_x - \hat{P}_y)\frac{t_x\sigma_x + t_y\sigma_y}{\lambda\theta.}. \tag{30}$$

In an economy with no distortions or frictions, the two measures of factor intensity (in terms of value and physical quantities) have the same sign so that $\lambda\theta > 0$. Thus the relative supply curve is always upward sloping.

Combining (24) and (28), we can see the fundamental difference caused by factor market frictions:

$$\hat{X} - \hat{Y} = (\hat{P}_x - \hat{P}_y)\frac{q_a - q_b + t_y\sigma_y}{\lambda(\theta - \Sigma)}. \tag{31}$$

It is now clear that the relative supply curve need not be upward sloping. Even if we could demonstrate that $\lambda\theta > 0$ (which need not be the case since the equilibrium is distorted), it would still be possible to have $\lambda(\theta - \Sigma) < 0$ and hence a downward-sloping relative supply curve. In the next section, we demonstrate that our economy is quite regular in the sense that when supply is downward sloping, sector X must be relatively small. In addition, by examining the sector X production process in detail, we are able to expose the forces at work that may lead to such perverse supply responses.

IV. Interpretation

We noted in Section II*E* that the search sector is generally too asymmetric. This production inefficiency is the source of the potentially perverse supply response derived in Section III. To see this, note that the fundamental difference between our search model and the standard general equilibrium model is the relationship between commodity and factor prices. When factor prices increase, there are two effects: commodity prices must rise as well if the firms are to continue to break even; in addition, firms will economize on the factor whose *relative* price has risen. In the standard model, this second effect is

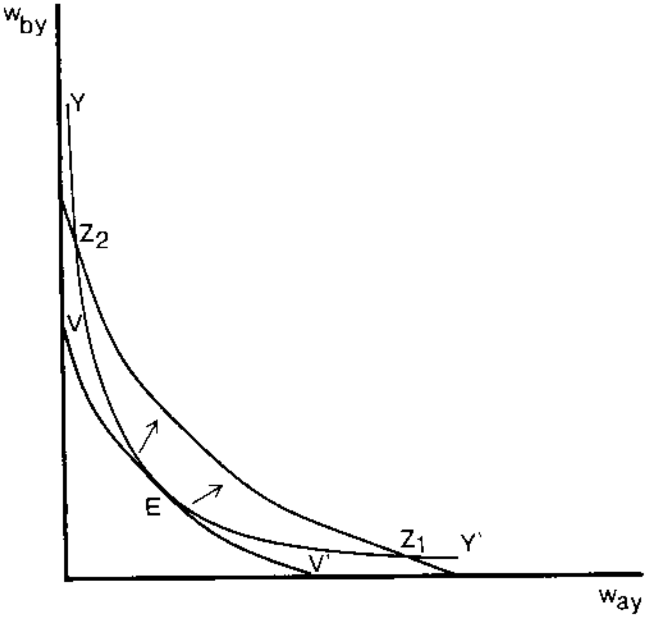

FIG. 1

zero by the envelope theorem. The introduction of factor market frictions causes this effect to be nonzero and to be important since the original equilibrium factor intensities are not optimal. An increase in w_a/w_b causes P_x/P_y to rise in the standard model if sector X is relatively A-intensive. With frictions, this effect may be offset if the new production technique adopted in sector X is more efficient, allowing P_x/P_y to actually fall. The purpose of this section is to determine the circumstances under which this might occur.

A. Equilibrium Factor Intensities

We begin by describing how the equilibrium physical factor intensities are determined and how they respond to changes in commodity prices. This is accomplished by focusing on the zero-profit condition for sector Y and the condition that guarantees indifference of workers across sectors.

Figure 1 is analogous to a factor price frontier. The YY' curve represents combinations of w_{ay} and w_{by} that are consistent with zero profits in sector Y. Formally, this curve is implicitly defined by equation (12), where l_{ay} and l_{by} minimize the cost of producing one unit of Y. Its slope is the physical factor intensity of the sector, $-l_y$.

Points along VV' represent combinations of w_{ay} and w_{by} consistent with equal expected lifetime incomes across sectors for both types of workers. Points above (below) VV' generate higher expected lifetime income in sector Y (X). To derive this curve, substitute the value of α_a given in (9) into (6) and then equate V_{is} and V_{iy} (see eq. [3]). We then obtain

$$w_{iy} = \frac{e_i P_x}{2 - r(2 - e_a - e_b)} \quad \text{for } i = a, b. \tag{32}$$

Given product prices and the proportion of searchers who are of type A (and thus e_i), (32) describes one point on VV'. As s increases, w_{ay}/w_{by} falls, and there is a movement up VV' to the left. We show in result A2 of the Appendix that the slope of VV' is $e_b[2(1 - r)\phi_b - re_a\phi]/e_a[2(1 - r)\phi_a + re_b\phi] < 0$.

Since both equilibrium conditions are satisfied at E, this wage vector is consistent with diversified production. However, at all other points along YEY', sector Y is able to offer high enough wages to draw all workers away from sector X and still break even. Therefore, except at E, production is specialized to good Y (see n. 9).

An increase in P_x (with P_y held constant) makes sector X more attractive and causes VV' to shift to the right. This curve now intersects YY' twice.[14] Both points of intersection represent a wage vector consistent with diversified production; yet one point (Z_1) represents an increase in w_{ay}/w_{by} while the other point (Z_2) signifies a decrease. Moreover, s falls (rises) as we move to Z_1 (Z_2) since w_{ay}/w_{by} and s are inversely related.

Once wages have been determined, we can derive the physical factor intensities. In sector Y, cost minimization requires $Y_a/Y_b = w_{ay}/w_{by}$. This equation defines l_y for any given vector of factor prices. Therefore, an increase in P_x causes l_y to decrease at Z_1 and to increase at Z_2.

Next, from (16), $l_x = [1 - r(1 - e_a)]e_b/[1 - r(1 - e_b)]e_a$, which is increasing in s (result A3 of the Appendix). This implies that an increase in P_x causes l_x to fall at Z_1 and to rise at Z_2. We conclude that increasing P_x causes both sectors to economize on factor A (B) if we are at Z_1 (Z_2).

B. Output Responses

The response of output to changes in relative commodity prices can be determined with the aid of figure 2. Suppose that the initial relative price (P) is such that there is a unique wage vector for sector Y

[14] Since VV' does not necessarily possess any nice curvature properties, there may be more than one point of tangency with YY' and more than two intersections when P rises. We restrict attention to the simplest case in this paper.

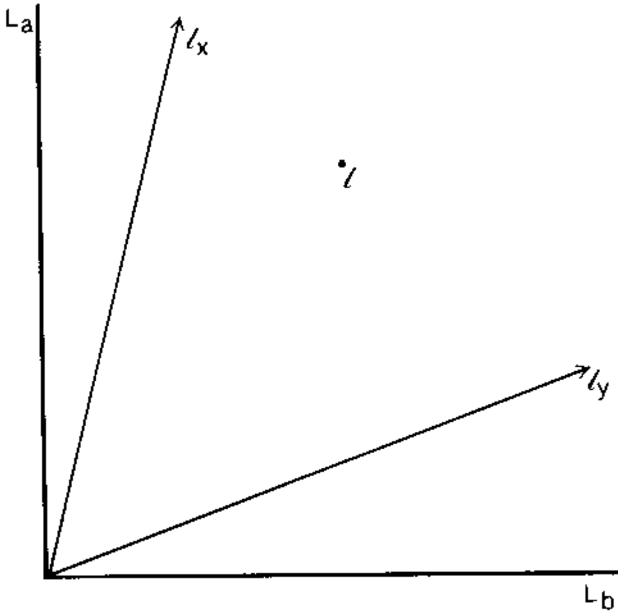

FIG. 2

consistent with diversified production (E in fig. 1). Suppose further that for any wage vector sector X is physically more A-intensive than sector Y. The factor intensities at E are depicted in figure 2. Finally, suppose that the economy's factor endowment is represented by l, which lies in the diversification cone.[15] We begin by noting that a drop in the relative price of X causes the economy to specialize in the production of Y. This follows since the fall in P causes VV' to shift down so that it does not intersect or touch YY'. In this situation, all firms produce Y since they can easily afford to pay the wages necessary to attract sector X workers. Therefore, P is the lowest relative price consistent with diversified production.

Now, suppose that P rises. This leads to two possible wage vectors for sector Y. If Z_1 represents the new equilibrium, both sectors become more B-intensive, causing the diversification cone to rotate counterclockwise. This implies that the demand for type B workers

[15] It is, of course, possible that l lies outside of the diversification cone when we are at E. If $l > l_x > l_y$ when VV' and YY' are just tangent, the economy is specialized to the production of good X. Increasing the relative price of good X will ultimately result in diversified production but with a negatively sloped relative supply curve throughout. If $l < l_y < l_x$ when VV' and YY' are just tangent, the economy is specialized to the production of Y. Increasing the relative price of good X will ultimately result in diversified production and a positively sloped supply curve throughout.

increases while the supply remains the same. To bring the demand for labor back into line, production much shift to the A-intensive good. Thus the production of X relative to Y rises.

This is not the case, however, if the new wages are given by Z_2. In this case, as P rises both sectors become more A-intensive. Equilibrium in the labor market therefore requires expansion of the sector that is relatively B-intensive. Thus the production of X falls as its relative price rises. Had we assumed that sector X was B-intensive, the result would have been reversed, with the type Z_1 equilibria leading to the perverse supply responses.

We are now in a position to relate this discussion to the algebra of Section III. At E of figure 1, YY' and VV' are tangent. If we equate the slopes derived above and use the fact that in equilibrium $e_b/e_a = w_{by}/w_{ay}$, we can show that, at E, $\Sigma = \theta$ (result A4 of the Appendix). In fact, whenever VV' is steeper (flatter) than YY', it can be shown that $\Sigma < (>) \theta$. Therefore, if sector X is A-intensive ($\lambda > 0$), then the type Z_1 equilibria are well behaved since $\lambda(\theta - \Sigma) > 0$. At Z_2, on the other hand, $\lambda(\theta - \Sigma) < 0$, implying that the supply response is perverse.

C. *Search Sector Production Functions*

To gain further insight into the forces behind our results, we devote this subsection to an analysis of the sector X production process. We do so because the fact that production in this sector is not technologically efficient leads to the downward-sloping portion of the supply curve.

The total number of type i workers in this sector at the end of the matching process is $L_{ix} = l_{ix}X$. Substitution from (16) yields

$$X = \frac{L_{ix}e_i}{1 - r(1 - e_i)}. \tag{33}$$

By varying s, we can obtain all combinations of L_{ax} and L_{bx} that produce X units of the search sector good. That is, we can obtain the search sector isoquant. By solving the L_{ax} expression for s, substituting into the L_{bx} expression, and differentiating, we can obtain the slope of this isoquant. Straightforward calculations yield $dL_{bx}/dL_{ax} = e_a\phi_b/e_b\phi_a < 0$. To guarantee that the isoquants are convex, we assume that $e_i''(s) < 0$.

Production efficiency requires that workers be distributed across sectors such that the marginal rates of technical substitution are the same in each sector. The slope of the sector Y isoquant is given by $-(Y_a/Y_b)$. Therefore, production efficiency requires that

$$\Omega = \frac{Y_a}{Y_b} + \frac{e_a\phi_b}{e_b\phi_a} = 0. \tag{34}$$

This condition will be met by the market only if, in equilibrium, the search sector is perfectly symmetric ($s = \frac{1}{2}$).[16] To see this, note that we have already shown that in equilibrium $Y_a/Y_b = w_{ay}/w_{by} = e_a/e_b$. The first equality follows from cost minimization in sector Y while the second follows from the worker indifference condition. Substituting into (34) yields

$$\Omega^e = \frac{e_a}{e_b} + \frac{e_a\Phi_b}{e_b\Phi_a}, \tag{35}$$

where the superscript e denotes that we are evaluating the term at equilibrium values. Since the search technology is symmetric, $\Omega^e = 0$ if and only if $s = \frac{1}{2}$. If $s \neq \frac{1}{2}$, then it is always possible to increase the production of Y while holding X constant by making sector X more symmetric (see Davidson et al. [1987] for details).

To see why this is important, return to figure 1. Note that as P_x rises, s falls in a type Z_1 equilibrium and rises in a type Z_2 equilibrium. Unless s is $\frac{1}{2}$ at point E, this implies that increases in P_x enhance the efficiency of sector X production in one type of equilibrium and hamper it in the other.

Suppose, for example, that sector X is physically more A-intensive for all wage vectors so that there are no factor intensity reversals. When this is true, s must be greater than $\frac{1}{2}$ at E (result A5 in the Appendix). Now suppose that we move to Z_1. As we do so, P_x/P_y and w_{ay}/w_{by} both rise as in the traditional model. The increase in w_{ay}/w_{by} causes P_x to rise relative to P_y since X is relatively A-intensive. This traditional effect is captured by θ. At the same time, however, s falls and approaches $\frac{1}{2}$, which implies that X is being produced more efficiently. This effect, captured by Σ, puts downward pressure on P_x/P_y. At Z_1 we know from above that the traditional effect outweighs the efficiency effect (since $\theta > \Sigma$). Hence, the mechanism linking movements in relative commodity prices and factor prices is qualitatively identical to the mechanism at work in the two-sector Walrasian model.

Now suppose that we move from point E to Z_2. As we do so, P_x/P_y rises and w_{ay}/w_{by} falls. The reduction in relative factor prices puts downward pressure on the relative price of good X since it is relatively A-intensive. As before, this traditional effect is captured by the θ term in (31). However, as we move toward Z_2, s rises and moves further from $\frac{1}{2}$. This implies that X is being produced less efficiently and puts

[16] The result that the optimal value of s is $\frac{1}{2}$ is an artifact of the assumption of symmetry in the matching function. If we let s^* denote the value of s that solves (34)—i.e., $\phi_a(s^*) + \phi_b(s^*) = 0$—then it can be shown that, in general, s^* is unique. Our results easily generalize to nonsymmetric matching functions if we then define "too asymmetric" to mean "too far from s^*." In addition, throughout the remainder of the text, phrases such as "s moves toward $\frac{1}{2}$" would need to be replaced by "s moves toward s^*."

upward pressure on P_x. This effect is captured by Σ, and at Z_2 it is this effect that dominates. Therefore, commodity and factor prices are no longer linked in the traditional manner, and it is this result that leads to the perverse supply responses. Again, had we assumed that X was relatively B-intensive ($\lambda < 0$) for all factor prices, then a similar argument could have been used to show that the perverse supply responses occur at type Z_1 equilibria.

Finally, we note that the perverse supply responses occur when the search sector is relatively *small.* It is as if the efficiency effect is subject to diminishing returns as the sector grows. Thus Σ dominates θ only when sector X is small. This is somewhat surprising since the only difference between this model and the Walrasian model is the inclusion of the trading frictions in one sector. One would therefore expect that if this sector were small, the model would continue to possess properties similar to those in the standard model. However, this is not the case.[17]

V. Applications

The formulation and analysis of a general equilibrium model with frictional unemployment serve two purposes. First, because we have been able to exposit the model in a familiar framework, one can easily examine a host of traditional issues in a framework that is somewhat more secure against the charge of unrealism to which any standard, full-employment Walrasian model is subject. It seems appropriate to address such questions within a model as similar as possible to those that economists have used in the past, for in this way one facilitates comparison with existing work and builds on earlier understanding. Second, by expanding the standard model in this manner we can examine questions that bear directly on issues surrounding unemployment. The examples below illustrate the usefulness of this framework.

A. *Tax Incidence*

In this subsection, we consider the impact of a partial factor tax on the earnings of type A individuals in sector X. Let $T_{ax} = 1 + t_{ax}$, where t_{ax}

[17] It is well known that perverse supply responses can occur when factor markets are distorted (see, e.g., Jones 1971, Magee 1976). Since the search sector production process in our economy is distorted, one might suspect that our results are caused by a factor market distortion. However, this is not the case. With only factor market distortions, reversal of the physical and value measures of factor intensity is both a necessary and a sufficient condition for generating perverse supply responses; this is simply not the case when frictions are introduced. In our working paper, we demonstrated that at E, the point at which the perverse supply responses begin, $\lambda\theta > 0$. For elaboration on this point see Davidson et al. (1986).

TABLE 1

	$s > 1/2$	$s < 1/2$
$\lambda > 0$	$\lambda\Sigma > 0$	$\lambda\Sigma < 0$
$\lambda < 0$	$\lambda\Sigma < 0$	$\lambda\Sigma > 0$

denotes the proportional tax rate. We assume that our economy is originally tax free and that all tax revenue is refunded in a lump-sum fashion to consumers.

The first step in the analysis is to add a demand side to the model. For simplicity, we assume that preferences are homothetic. In this case, it is well known that the demand side is represented by

$$\hat{X} - \hat{Y} = -\sigma_d(\hat{P}_x - \hat{P}_y), \tag{36}$$

where σ_d represents the elasticity of substitution in consumption.

Incorporating the partial factor tax into the supply side of our model yields the following basic equations (see result A6 in the Appendix):

$$\lambda(\hat{X} - \hat{Y}) = (\hat{w}_{ay} - \hat{w}_{by})(q_a - q_b + t_y\sigma_y) - (q_a - q_b)\hat{T}_{ax}, \tag{37}$$

$$\hat{P}_x - \hat{P}_y = (\theta - \Sigma)(\hat{w}_{ay} - \hat{w}_{by}) + (\alpha_a - \Sigma)\hat{T}_{ax}. \tag{38}$$

The algebraic expression for the incidence of T_{ax} is

$$\sigma(\hat{w}_{ay} - \hat{w}_{by}) = [-(q_a - q_b) - \sigma_d\lambda\alpha_a + \Sigma\lambda\sigma_d]\hat{T}_{ax}, \tag{39}$$

where $\sigma \equiv \lambda(\theta - \Sigma)\sigma_d + (q_a - q_b + t_y\sigma_y)$ corresponds to Jones's aggregate elasticity of substitution. We assume an adjustment process that directs factors to move between sectors in order to equalize returns. With this assumption, σ must be positive in order to guarantee local stability.

The incidence expression in (39) has three terms on the right-hand side. The first two are the usual factor substitution and output effects, but the last term captures the effect of the tax on relative wages through changes in the efficiency of the search process. This effect depends on the sign of $\lambda\Sigma$, which in turn depends on both the physical factor intensities and s, the mix of searchers. To see this, note that from the definition of Σ we have $\text{sign}(\Sigma) = \text{sign}(s - 1/2)$. The possibilities are summarized in table 1.

Regardless of the factor intensities, the initial impact of the tax causes sector X to contract. If sector X is relatively A-intensive ($\lambda > 0$), it contracts by adopting more A-intensive techniques, thereby increasing s. This increase in s enhances search efficiency only if the original s is less than $1/2$. If this is the case, the improved efficiency puts downward pressure on P_x/P_y, and the resulting exit of factors further augments the relative decline in returns to the factor intensively employed in the sector. Consequently, w_{ay} falls further relative to w_{by} in this case. If, on the other hand, s initially exceeds ½, the contraction of sector X is offset by the decline in search efficiency. The case in which sector X is B-intensive is analogous.

In addition to its impact on wages paid in sector Y, the factor tax can also be expected to change the ratio of expected lifetime income if currently matched to that if currently searching. From (6), (7), and (9) we have

$$\frac{V_{im}}{V_{is}} = \frac{1 - (1 - d)(1 - e_i)}{e_i}. \tag{40}$$

Differentiation of (40) yields

$$\hat{V}_{im} - \hat{V}_{is} = \left(\frac{\phi_i d}{e_i}\right)\hat{s}. \tag{41}$$

The effect of the tax on the relative welfare of the employed depends solely on its effect on the composition of the searchers. For example, if the tax increases s, type A searchers have a more difficult time finding a match, and thus the relative welfare of their employed counterparts rises.

Intuitively, the tax affects s in two ways. First, it causes sector X to contract. If X is relatively A-intensive, it must become more B-intensive as it contracts so that the conditions in (14) continue to hold. As X becomes more B-intensive, s falls. In addition to this output effect, there is a substitution effect since sector X becomes relatively less attractive to type A workers. In this case, the substitution effect reinforces the output effect, and s falls. If sector X is relatively B-intensive, the output effect would be reversed and would tend to offset the substitution effect.[18]

B. *The Protection of Jobs*

Calls for protection of domestic production against foreign competition are strongest when economic opportunities are lowest among a segment of the population. Wages and unemployment rates are two dimensions of these economic opportunities, and the question arises as to the effectiveness of, for example, tariff barriers in improving wages and reducing unemployment rates.

To address the consequences of protection, assume that sector X is relatively A-intensive, that the initial equilibrium is stable, and that X is imported. Now impose a small tariff that increases the relative

[18] Using (41) and result A6, we obtain $\sigma\hat{s} = (1/\phi)(-\lambda\sigma_d\theta_{ay} + t_y\sigma_y)\hat{T}_{ax}$. The first term in parentheses on the right-hand side represents the output effect, while the second term reflects the substitution effect.

domestic price of X. The first issue is the effect on the relative well-being of the protected factor, type A individuals, measured as V_{as}/V_{bs}, which in equilibrium equals relative wages. In essence, this is the standard Stolper-Samuelson discussion extended to the case of friction-ridden factor markets.

We can see from equation (28) that the impact of a change in relative prices on relative wages depends on the sign of $(\theta - \Sigma)$, which (because $\lambda > 0$) has the same sign as the slope of the relative supply curve. Thus if the initial equilibrium is on the upward-sloping portion of the relative supply curve (where $\theta > \Sigma$), small tariffs enhance the relative well-being of unemployed type A individuals. On the other hand, $\theta < \Sigma$, which occurs only when sector X is relatively small, implies a decline in the relative wage of the protected type. In general, the Stolper-Samuelson relationship depends on the mechanism linking relative product and relative factor prices. Along the downward-sloping part of the relative supply curve this relationship does not correpond to that implied by relative factor intensities.

In addition to its influence on wages, the tariff also affects the rate of unemployment, $\mu = (L_{as} + L_{bs})/(L_a + L_b)$. From (10) and (11), we obtain

$$\mu = (1 - r)\left[\left(\frac{1 - e_a}{e_a}\right) + \left(\frac{1 - e_b}{e_b}\right)\right] \frac{X}{L_a + L_b}. \tag{42}$$

Logarithmic differentiation of (42) reveals that

$$\hat{\mu} = -\left(\frac{e_b\phi_a + e_a\phi_b}{e_a + e_b - 2e_ae_b}\right)\hat{s} + \hat{X}. \tag{43}$$

The coefficient of $\hat{s}$ in (43) is monotonically increasing in s. Furthermore, the coefficient is negative, zero, or positive as s is less than, equal to, or greater than ½. Therefore, the unemployment rate varies inversely with the symmetry of sector X and directly with its size. There are two opposing effects on the unemployment rate. First, as s moves toward ½, the sector grows more symmetric and unemployment per unit of X declines. On the other hand, the sector itself increases in size, bringing more unemployment. We derive the effect of any tariff by noting that it shifts VV' away from the origin in figure 1. If the initial equilibrium is Z_1 (the relative supply slopes upward), then sector X expands and s falls. If, in free trade, $s < ½$, then unemployment rises unambiguously. Extension to the case involving Z_2 and $s > ½$ is straightforward.

C. *Minimum Wages*

The impact of minimum wage laws has been studied in a general equilibrium context by several authors including Johnson (1969) and

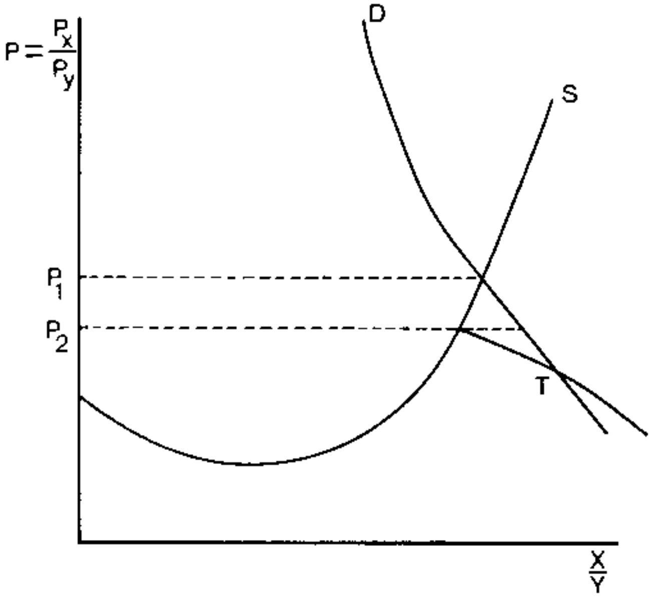

FIG. 3

Mincer (1976). Both authors emphasize that induced changes in the output mix due to minimum wages can significantly compromise partial equilibrium results. Our exposition of the simple general equilibrium model with frictional unemployment can address this issue in a context in which unemployment arises naturally and exists independently of the minimum wage. As we show, this feature also permits a useful treatment of the issue of voluntary and involuntary unemployment.

Consider the incidence of a global minimum wage. Again assume that sector X is relatively A-intensive in the physical sense. Let Y serve as the numeraire and assume that the initial, unconstrained equilibrium is on the upward-sloping portion of the relative supply curve as diagrammed in figure 3.

Figure 4 presents an initial supply-side equilibrium at Z_1. Notice that type B workers are the low-wage earners; thus a universally applicable minimum wage increases w_{by}, for this is the lowest wage paid in the economy. Let $\underline{w}$ represent the minimum wage. The wage for type A workers that permits zero profits in sector Y is then w'_{ay}, but the vector $(\underline{w}, w'_{ay})$ creates incentives for workers to emigrate to sector X, which gives higher expected lifetime income. As workers move to sector X, the output of X increases and its demand price falls, shifting VV' toward the origin until factor market equilibrium is restored at Z'_1. This is not an equilibrium for the economy, however, because at Z'_1

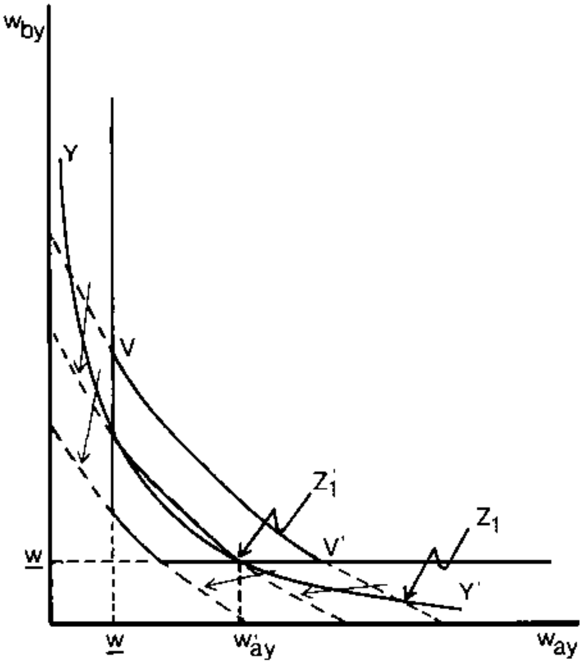

FIG. 4

the supply price falls short of the marginal willingness to pay of demanders. To see this, note that there is a price below P_1 in figure 3 that would clear factor markets at the wages indicated by Z_1'. At this price the minimum wage would not be binding; yet $w_{by} = \underline{w}$. We denote this price by P_2 in figure 3 and observe that the product market is not in equilibrium.

Since the relative demand price for X exceeds the relative supply price, workers attempt to move to sector X, but because sector Y must release factors in a constant, relatively B-intensive proportion (to maintain the wages at Z_1 in fig. 4), the proportion of type A workers in sector X falls. To determine the effect of this migration on price, we recall the worker indifference condition. Equation (13) holds for the unconstrained type A workers, but the analogous condition for type B workers is now $w_{by} = \underline{w}$. Thus w'_{ay} solves $c_y(w'_{ay}, \underline{w}) = 1$. Substituting from (6), (9), and (13) yields

$$w'_{ay} = \frac{e_a P}{2 - r(2 - e_a - e_b)}. \tag{44}$$

With w'_{ay} given, the decline in s must be compensated by a fall in P_x to leave type A workers indifferent between sectors. In figure 3, we show P_x/P_y *falling* as X/Y increases toward the ultimate equilibrium at

T. This fall in the relative price of X redounds on the factor markets by shifting VV' in figure 4 in further. The equilibrium wages remain at Z_1', where YY' intersects the horizontal portion of VV'. Type A workers are indifferent between sectors, but type B workers prefer sector Y. The minimum wage prevents these unemployed type B's from underbidding their counterparts employed in sector Y.

Without a minimum wage, there are four "classes" in the economy: type i searchers and their employed counterparts in sector Y, who earn equal expected income, and type i matched workers in sector X, who are unambiguously better off. The minimum wage creates a new lower class: those type B searchers who, although they strictly prefer employment in either sector, cannot find a job. The minimum wage is designed to improve the welfare of low-wage workers (it *does* raise the well-being of those who retain sector Y jobs); yet it condemns those unemployed to longer durations of unemployment (e_b declines with the decrease in s) and lower earnings when they do find sector X jobs (α_b shrinks as the bargaining strength of type B workers diminishes).

This restriction on low wages has implications for the issue of voluntary and involuntary unemployment. In the unconstrained equilibrium, one can reasonably argue that all observed unemployment is voluntary. If an unemployed worker were given the choice of taking a job in sector Y or continuing to search for a job in sector X, she would be indifferent. With the minimum wage, type A workers continue to maintain indifference across sectors and thus can be construed to choose their unemployment voluntarily. On the other hand, there is an excess supply of type B workers to sector Y. Those who cannot find employment at $\underline{w}$ are forced to seek employment in sector X. If asked, a type B worker who is searching in sector X would strictly prefer to work in sector Y at the going wage, but no jobs are available. All unemployed type B workers are, in a very real sense, *involuntarily* unemployed.

VI. Conclusion

There is no doubt that over the years the use of the simple two-sector general equilibrium model has led to many valuable insights. Notwithstanding its wide applicability, many issues require the development of new models. For example, Diamond's general equilibrium search models (1984*a*, 1984*b*) provide us with a framework in which to consider traditional macroeconomic questions (e.g., the neutrality of money and the effects of government policies on business cycles) in a microeconomic framework. Our intent in this paper is to reconcile these new models with the existing body of literature concerning simple general equilibrium models. This approach facilitates comparison and discerns how much of the earlier intuition is preserved. This gives us some idea as to what extent our analyses and conclusions need to be modified. Toward this end, we have provided a model that is very much in the spirit of the simple two-sector general equilibrium model yet allows for frictional unemployment. We then analyzed its structure in a manner that encourages comparison with Jones (1965). We demonstrated that while much of the structure remains, the addition of frictions in the labor market alters the basic relationship linking factor and commodity prices. This new relationship admits the possibility of downward-sloping relative supply curves, especially when the search sector is small. We then used our model to reexamine three old questions: the incidence of taxation, the effects of protection, and the impact of minimum wage laws. We demonstrated that our synthesis can yield new insights in each case.

Appendix

Result A1

Begin by using (3), (6), (13), and (16) to obtain

$$\alpha_i P_x = \left[\frac{1 - r(1 - e_i)}{e_i}\right] w_{iy} = l_{ix} w_{iy}.$$

Now sum $\alpha_a P_x$ and $\alpha_b P_x$ to obtain (15).

Result A2

Two equations define the VV' curve: $V_{iy} = V_{is}$ for $i = a, b$. From (6) and (3) these equations imply

$$w_{iy} = \frac{e_i P_x}{2 - r(2 - e_a - e_b)}.$$

Using the w_{ay} expression we could, in principle, solve for s as a function of w_{ay}. Substituting this function into the w_{by} expression then yields the VV' curve. Define $G(w_{by}, s(w_{ay})) = 0$ to be the equation for VV'. Totally differentiating G we obtain

$$\frac{dw_{by}}{dw_{ay}} = -\frac{(\partial G/\partial s)(\partial s/\partial w_{ay})}{\partial G/\partial w_{by}}.$$

Now use the implicit function theorem to obtain $\partial s/\partial w_{ay}$ from the w_{ay} expression above and substitute to obtain

$$\frac{dw_{by}}{dw_{ay}} = \frac{e_b'[2(1 - r) + e_a r] - e_a' e_b r}{e_a'[2(1 - r) + e_b r] - e_b' e_a r}.$$

Straightforward algebra can now be used to obtain the desired expression.

Result A3

We wish to show that l_x is an increasing function of s. We have

$$l_x = \frac{[1 - r(1 - e_a)]/e_a}{[1 - r(1 - e_b)]/e_b}.$$

The derivative of the numerator of this expression is

$$\frac{re_a'e_a - [1 - r(1 - e_a)]e_a'}{e_a^2} = -\frac{e_a'(1 - r)}{e_a^2} > 0.$$

By symmetry, the denominator is decreasing in s. Therefore, l_x is increasing in s.

Result A4

We wish to show that, at point E, $\Sigma = \theta$.

At point E the slope of YY' is $-l_y = -(l_{ay}/l_{by})$ and the slope of VV' is $e_b[2(1 - r)\phi_b - re_a\phi]/e_a[2(1 - r)\phi_a + re_b\phi]$. Equate these values, multiply both sides of the equation by e_a/e_b, and substitute w_{ay}/w_{by} for e_a/e_b to obtain

$$-\frac{\theta_{ay}}{1 - \theta_{ay}} = \frac{2(1 - r)\phi_b - re_a\phi}{2(1 - r)\phi_a + re_b\phi}.$$

Solving for θ_{ay} yields

$$\theta_{ay} = \frac{e_a r\phi - 2(1 - r)\phi_b}{\phi[2 - r(2 - e_a - e_b)]}.$$

Now $\theta = \alpha_a - \theta_{ay}$ and α_a is given in (9). Combining the two yields Σ.

Result A5

We wish to demonstrate that $s > ½$ at point E if and only if $\lambda > 0$. Now begin by noting that $\lambda = l_x - l_y$, where l_x is given in result A3 and, at point E, $-l_y$ is equal to the slope of the VV' curve, which is given in result A4 above. Substituting these values into λ and subtracting yields

$$\text{sign}(\lambda) = \text{sign}[-(\phi_a + \phi_b)].$$

To sign $\phi_a + \phi_b$, simply use our assumption that $e_i''(s) < 0$. Finally, note that by symmetry of the search technology, $\phi_a + \phi_b = 0$ when $s = ½$.

Result A6

We define T_{ix} such that the net income of a matched worker is $w_{ix} = (\alpha_i P_x)/T_{ix}$. The expected consumption resulting from a match is $\alpha_i P_x/(1 - r)T_{ix}$. Thus, from (4) and (5), we have the surplus due to a match:

$$\frac{\alpha_i P_x}{(1 - r)T_{ix}} - \frac{(1 - d)(1 - d - r)}{1 - r} V_{is}.$$

Since only type A income is taxed ($T_{bx} = 0$), the Nash bargaining solution does not split the surplus evenly. The solution requires maximization of the product of the surpluses. The first-order conditions to this problem imply

$\alpha_a/(1 - \alpha_a) = [1 - r(1 - e_a)]/[1 - r(1 - e_b)]$. Substituting into (13) and dividing the equation for $i = a$ by that for $i = b$ yields $e_a/e_b = w_{ay}T_{ax}/w_{by}$. Differentiating, we obtain $\phi\hat{s} = \hat{w}_{ay} - \hat{w}_{by} + \hat{T}_{ax}$. Repeating the derivations in (21)–(29) yields (37) and (38).

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