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SEARCH, UNEMPLOYMENT, AND THE PRODUCTION OF JOBS*

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The purpose of this paper is to present and analyse a simple two sector general equilibrium model which incorporates some of the real world frictions that keep factor markets from functioning perfectly. In particular, we investigate the performance of the economy when search is required to find employment and when the duration of unemployment varies across sectors. This is accomplished by assuming that in one sector, factor markets are frictionless so that the duration of unemployment is zero, while in the other sector idle factors of production must actively search each other out in order to produce. These factors are mobile across sectors and therefore distribute themselves such that, in equilibrium, the expected lifetime return to each of the two factors is the same in both sectors.

In solving for the steady state equilibrium, we pay particular attention to how well the economy allocates workers *across* sectors and to conditions under which the factor mix *within* a sector is correct. It is by now well established that when search is required to find trading opportunities, externalities are generated that usually lead to inefficiencies. For example, Diamond (1982, 1984) and Pissarides (1984*a*) have demonstrated that these externalities lead to an equilibrium level of search that is likely to be too low.[1] The Pissarides and 1982 Diamond papers, however, are partial equilibrium approaches that take the number of market participants as exogenously given.[2] Therefore, they cannot address the types of issues in which we are interested. On the other hand, the 1984 Diamond book follows a general equilibrium approach, but includes only one sector and one factor of production. This makes it impossible to analyse the optimality of the output and factor mixes. Moreover, to investigate the implications of unequal durations of unemployment across sectors, the minimal requirement is a two sector general equilibrium model.

The externalities that play such a prominent role in the Diamond and Pissarides studies lead to inefficiencies in our model as well. For example, we demonstrate that only in rare cases will the economy operate along its pro-

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[1] In these studies (and ours as well), search intensities are not allowed to vary. In this context, when we say that a worker does not search enough we mean, for example, that the worker's reservation wage is too low for efficiency. For studies that investigate the effect that these externalities have on variable search intensities, see Mortensen (1982*a*, *b*) and Pissarides (1984*b*). In addition, production in our model is deterministic (once a job is created); see Pissarides (1984*a*) or Jovanovic (1984) for search models with randomness in the production process. For recent surveys of the search literature see Hosios (1986) and Mortensen (1986*a*).

[2] This is also true of the Mortensen and Pissarides papers listed in the first note.

duction possibilities frontier. In addition, we show that the search sector is too small and its factor intensity is too asymmetric. Holding the factor intensities constant, the number of steady state searchers increases as the sector grows, therefore increasing unemployment. On the other hand, we show that as this sector becomes more symmetric, the number of steady state jobs increases, thereby reducing the unemployment rate. Equilibrium unemployment may therefore be either too high or too low.

An interesting feature of our model that has not been emphasised in previous work concerns the value of jobs. Many economists would agree that if we ignore equity considerations, the value of a job in a competitive economy is fully measured by the amount of output it produces. This is not true of the jobs held in the search sector of our model. The reason for this is actually rather simple. Employed workers at time t experience higher utility than their unemployed counterparts for two reasons: (*i*) current consumption is greater; and (*ii*) the probability of being employed in $t+1$ thenceforth is higher. The value of the former is exactly the value of output, because all search sector output is distributed to employed workers. The extra value of the employment is the expected additional consumption for the life of the job. Each job therefore creates a joint product – a unit of output today and a stream of future output at a lower cost. Steady state welfare is measured by the value of output *plus* the value of jobs in the search sector.

Once we have established that the value of jobs belongs in the welfare function, we go on to consider whether production efficiency is a necessary condition for Pareto optimality. In standard models, the sectoral factor mix which is Pareto optimal is the one which minimises the opportunity cost of production of that sector's output *measured in terms of foregone output of the other sector*. When jobs are at issue, however, there is a second role for the factor mix in that it alters the value of those jobs. For example, the value of a job increases as the probability of successfully finding employment falls. We show that for a certain class of search technologies, the value of a given number of jobs is independent of the factor mix and thus production efficiency is optimal. However, this is not true in general.

We outline the model in the next section and proceed to the determination of equilibrium in Section II. We provide an analysis of the efficiency properties of equilibrium in Section III, distinguishing between allocative and production efficiency. In Section IV we discuss some characteristics of the social optimum. First, we compare the magnitude of equilibrium versus optimal unemployment. Second, we demonstrate that productive efficiency may not be a necessary condition for Pareto optimality, depending upon the search technology. Finally, we offer some concluding remarks in Section V.

I. THE MODEL

Our economy consists of two sectors, X and Y, and two types of agents, A and B, each endowed with one indivisible unit of leisure which they may either consume or offer as labour. Agents are finitely lived, facing a constant non-zero probability of death each period. The birth and death rates for each type of agent are assumed equal and are denoted by τ so that we have zero population growth.

In each period, the agents choose a sector in which to seek employment. In the Y sector, output is produced according to a neoclassical, constant returns to scale production function:

$$Y = Y(R_a, R_b), \tag{1}$$

where Y is the total output of this sector and R_i is the amount of type i labour employed. All markets in this sector are assumed to be frictionless and perfectly competitive, so that workers in sector Y are never unemployed and always earn the value of their marginal product. If we let w_i denote the wage paid to a type i worker in this sector and choose good Y to be the numeraire, then this last condition can be expressed as

$$Y_i = \frac{\partial Y}{\partial R_i} = w_i. \tag{2}$$

Individuals may also choose to seek employment in sector X. In this sector, production of output requires an agent to find a partner of the opposite type. This is accomplished by searching each period until a suitable partner is found. In each period that the agent fails to find a partner, she remains idle and earns no income. When an agent of the opposite type is found, a partnership begins which results in the production of one unit of output each period as long as both partners survive. When a worker's partner dies, she must begin searching again.

We assume that the revenue generated by the output produced in the search sector is distributed between the partners according to the Nash cooperative bargaining solution. This solution splits the surplus created by the match evenly.[3] For future use, we let θ_i denote the share of output that goes to a type i agent. Of course, $\theta_a + \theta_b = 1$. Finally, we refer to a partnership as a 'match' or 'job', and the total amount of output produced each period (X) is equal to the number of agents of each type who are matched or employed (M).

The ease with which a worker can find a job in the search sector depends upon the search technology inherent in the economy. In our model, this technology is represented by two functions, $\pi_i(\lambda)$ for $i = A$, B, that give the per period probability of a type i worker finding a match as a function of λ, the proportion of agents searching at the *beginning* of the period who are type A workers. In order to simplify the analysis, we place several restrictions on these functions. First, we assume that as the proportion of searchers who are of type A increases, it becomes more difficult (easier) for a type A (B) agent to find a suitable partner. We therefore assume that $\pi'_a(\lambda) < 0$ and $\pi'_b(\lambda) > 0$. Second, by writing this probability as only a function of the *proportion* of searchers of

[3] Recent developments in bargaining theory justify the use of the Nash cooperative bargaining solution. A natural bargaining process that can be envisaged involves agents exchanging sharing rules until one agent makes an offer that is acceptable to her partner. If we assume that the time it takes to make counter-offers is very small (i.e. arbitrarily close to zero), then we know that the outcome of this non-cooperative game is identical to the Nash cooperative bargaining solution. For details, see the work of Rubinstein (1982), Binmore (1982), McLennan (1982), and Rubinstein and Wolinsky (1985).

each type as opposed to the *number* of searchers of each type, we are assuming constant returns to scale in the search technology.[4] Finally, we assume that the search technology is symmetric. That is $\pi_i(\lambda) = \pi_j(1-\lambda)$.[5]

To summarise to this point, in every period the same five step procedure repeats itself. First, all workers who were matched at the end of the previous period and who have survived wake up and check to see if their partner has survived as well. Second, unmatched workers decide in which sector to seek employment. Third, all those seeking employment in sector X search for partners. Fourth, bargaining between the new partners takes place. Fifth, workers produce, divide, and consume the output.

We are now in a position to examine the worker's choice of a sector in detail. To do so, we introduce the following additional notation:

p = the relative price of X in terms of Y;

V_{is} = the expected value of lifetime income to a type i worker who is searching in the X sector;

V_{im} = the expected value of lifetime income to a type i worker who is currently matched in the X sector;

V_{iy} = the expected value of lifetime income to a type i worker who is employed in the Y sector.

We assume that both types of agents are risk neutral.[6] In this case, workers evaluate the potential for entering each sector based on the expected lifetime income generated by entering one sector or the other. If sector X offers type i agents greater expected lifetime income than sector Y, then all type i agents will flow to sector X. Production can be diversified only if entry into both sectors generates the same expected lifetime income for an agent of type i.

For the Y sector, expected lifetime income simply depends upon the wage in that sector and the death rate of workers. This follows since workers in this sector are never unemployed. Since a type i worker in sector Y earns w_i each period, V_{iy} is given by

$$V_{iy} = \frac{w_i}{r}. \tag{3}$$

[4] Empirical support for this assumption is provided by Nickell (1979).

[5] An alternative way of modelling the search technology is to begin with a function that describes the creation of new matches. We can write this function as $N = N(S'_a, S'_b) \leqslant \min(S'_a, S'_b)$, where N is the number of new matches created and S'_i is the number of type i agents who *begin* the period searching for employment in the X sector. If we assume that $N(\ )$ is concave, symmetric, increasing in both arguments, and exhibits constant returns to scale, then the properties of $\pi_i(\lambda)$ follow naturally. To see this, note that π_i, the probability that a randomly chosen type i searcher becomes matched is equal to N/S'_i. Dividing the numerator and denominator by the total number of searchers $(S'_a + S'_b)$ permits us to express π_i solely as a function of λ, the proportion of searchers who are type A agents (i.e. $\lambda = S'_a/(S'_a + S'_b)$). Next, symmetry follows because

$$\pi_a(\lambda) = \frac{N(\lambda, 1-\lambda)}{\lambda} = \frac{N(1-\lambda, \lambda)}{\lambda} = \pi_b(1-\lambda).$$

Finally,

$$\pi'_a(\lambda) = \frac{1}{\lambda^2}(\lambda N_1 - \lambda N_2 - N) = \frac{-N_2}{\lambda^2} < 0.$$

The last equality follows from the assumption of constant returns to scale.

[6] This assumption, quite common in the search literature, highlights the fact that the unemployment that emerges from the model is not the result of optimal risk sharing arrangements, as in Azariadis (1975) and Baily (1974). Rather, the unemployment stems from the frictions in the labour market that make it difficult for agents to find each other.

The expected lifetime income for a type i searcher can be written as

$$V_{is} = \pi_i(\lambda)\, V_{im} + [1-\pi_i(\lambda)](1-\tau)\, V_{is}. \quad (4)$$

In words, expected lifetime income for a worker currently searching in sector X is a weighted average of expected income if matched and expected income if not matched, where the latter is the expected income of the remainder of life if one lives through the idle period.

The expected lifetime income if matched is

$$V_{im} = \theta_i p + (1-\tau)^2 V_{im} + \tau(1-\tau)\, V_{is}, \quad (5)$$

where the first term is the first period earnings, the second is the expected value of the match should both partners survive the period, and the third is the expected value of income for the remainder of life should one partner die. Equations (4) and (5) can be solved to obtain

$$V_{is} = \pi_i \theta_i p/\tau[1-(1-\tau)^2(1-\pi_i)], \quad (6)$$

$$V_{im} = [1-(1-\tau)(1-\pi_i)]\, \theta_i p/\tau[1-(1-\tau)^2(1-\pi_i)], \quad (7)$$

where the argument of π_i has been suppressed for notational convenience.

Since V_{is} and V_{im} both depend on θ_i, we now provide an explicit description of the solution to the bargaining problem. As indicated above, we employ the Nash cooperative bargaining solution, which divides the surplus created by a match evenly. The payoff to a type i agent from a match is V_{im} and the threat point is $(1-\tau)\, V_{is}$, as the worker can withdraw from the bargaining and then search again next period if she survives. Therefore, the surplus generated for a type i worker is

$$V_{im} - (1-\tau)\, V_{is} = \theta_i p/[1-(1-\tau)^2(1-\pi_i)]. \quad (8)$$

Equating $V_{am} - (1-\tau)\, V_{as}$ with $V_{bm} - (1-\tau)\, V_{bs}$ and solving for θ_a, we obtain

$$\theta_a = [1-(1-\tau)^2(1-\pi_a)]/[2-(1-\tau)^2(2-\pi_a-\pi_b)]. \quad (9)$$

For given values of p and τ, (9) defines θ_a as a function of λ. Furthermore, it is straightforward to show that the two variables are inversely related. The reason for this is clear; if the type A workers are relatively abundant in the X sector, they are in a weaker bargaining position and can expect to come out of the bargaining process with a smaller share of output.

Finally, we assume that the total steady state populations of A and B types are fixed exogenously. After the matching process is completed, an individual is either matched in the X sector, still searching in the X sector, or working in the Y sector. Therefore, we have two identities

$$N_i = M + S_i + R_i \quad (i = A, B), \quad (10)$$

where N_i denotes the number of type i workers and S_i denotes the number of type i searchers *after* the matching process is completed.[7]

[7] Note that because of the discrete time nature of the model, S_i does not equal S'_i, the number of type i agents who *begin* the period searching. As indicated in note 5 on p. 860 it is the latter that is relevant for determining the probabilities of a match.

The supply side of the model is completely specified by equations (1)–(10). The demand side could be added by writing demands for X and Y as functions of income and relative prices. For simplicity, however, we assume instead that we are dealing with a small open economy so that we may treat p as an exogenous variable. All of our results hold for all p for which both goods are produced; thus there is no loss of generality in doing so.

II. EQUILIBRIUM

The steady state equilibrium of this economy is characterised by the following conditions: (*i*) steady state in the X sector; (*ii*) indifference between sectors for both types of workers (to ensure positive output levels of both goods); (*iii*) zero excess demand for labour of each type in the Y sector; and (*iv*) zero profits in the Y sector (since we have assumed constant returns to scale and competitive behaviour).

In a steady state, the number of each type who are matched and working in the X sector remains constant over time. This condition is given by

$$M = (1-\tau)^2 M + (1-\tau)\,\tau\pi_i M + \beta_i \tau N_i \pi_i + S_i(1-\tau)\,\pi_i, \tag{11}$$

where β_i denotes the proportion of the newly born type i workers who choose to seek employment in sector X. One can compute the total number of type i individuals who are matched in period t by examining how individuals might arrive in the matched state from the previous period $t-1$. The first term on the right hand side is the number who were matched at the end of the previous period, lived and had a partner who survived as well. The second term reflects those who were matched, lived, had a partner who died, and yet were able to find a new match immediately. The third term represents those who were born, went to the X sector, and immediately found a match. Finally, there are those who were searching at the end of the period, lived, and found matches.

In addition to (11), steady state also requires that

$$\tau(M+S_i) = \beta_i \tau N_i \tag{12}$$

which ensures that the number of agents who exit the sector equals the number who enter.[8] Using (11) and (12), we find that

$$S_i = \left(\frac{1-\pi_i}{\pi_i}\right)[1-(1-\tau)^2]\,M. \tag{13}$$

[8] The steady state conditions (11) and (12) combined with the symmetry of the matching technology guarantee that the value of λ determined in the model will be consistent with its definition. To see this, use (12) to substitute for β_i in (11) and then equate the two steady state equations in (11). Next, due to the symmetry of the matching technology (see note 5 on p. 860), we have

$$\frac{\pi_a(\lambda)}{\pi_b(\lambda)} = \frac{1-\lambda}{\lambda}.$$

Finally, substituting this value back into (11) and solving for λ we obtain

$$\lambda = \frac{S_a + [1-(1-\tau)^2]\,M}{2M[1-(1-\tau)^2] + S_a + S_b} = \frac{S_a'}{S_a' + S_b'}$$

which is the definition of λ (the last equality follows from note 10 on p. 866).

We turn next to the second equilibrium condition. Since workers of each type may seek employment in either sector, the expected lifetime income from entering either sector must be the same in equilibrium; that is

$$V_{is} = V_{iy} \quad (i = A, B). \tag{14}$$

Notice that (14) does not imply that average income is equal across sectors, since matched workers are necessarily better off than their unmatched counterparts. Individuals cannot, however, choose to be matched; they can only choose to search. The expected lifetime income for this choice is V_{is}, not V_{im}. Finally, note that whenever (14) holds, we can be assured that the third equilibrium condition will hold. Since workers are indifferent between sectors, sector X will act as a residual, soaking up all workers not employed in sector Y.

Before turning to the last equilibrium condition, we wish to examine the equations in (14) in greater detail. If we add equation (9), which described the bargaining solution and substitute $\theta_b = 1 - \theta_a$, we now have three equations in four unknowns (w_a, w_b, θ_a and λ). According to these equations, higher wages for type A workers are associated with lower values of λ, while w_b varies directly with λ. That is, as the proportion of searchers of type A grows, the probability of finding a match falls for all type A searchers. Their reservation wage, which is the certainty equivalent wage, is therefore correspondingly lower. As λ approaches one, the chance of a type A worker finding a match goes to zero and searching in the X sector has a value equivalent to receiving a wage of zero with certainty. On the other hand, as λ falls to zero, finding a match becomes a certainty and the reservation wage equals $\theta_a p$. Similar reasoning explains the relationship between w_b and λ.

In Fig. 1 we derive the contour $\bar{U}$, which indicates the combinations of w_a

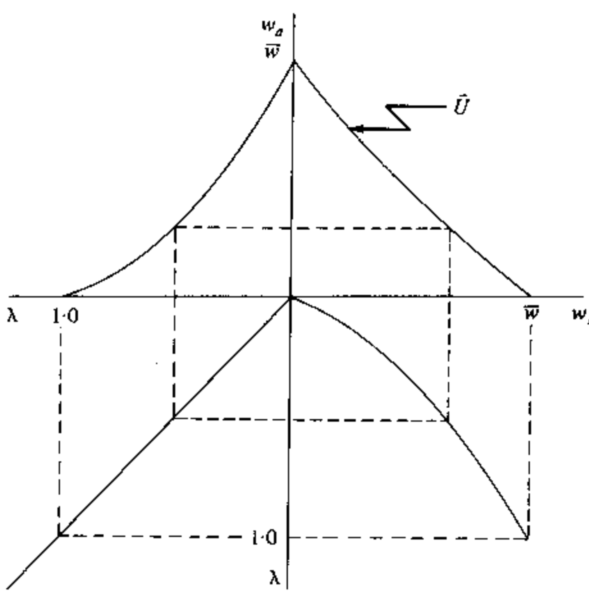

Fig. 1

and w_b that keep both populations indifferent between sectors. The contour $\bar{U}$ is somewhat analogous to an isocost curve for sector X. If w_a and w_b are too high, the (opportunity) cost of searching in the X sector is too large, and all agents will move to sector Y. Increases in p shift the $\bar{U}$ curve out from the origin since, as X sector rewards increase, Y sector wages must rise to restore indifference. The wages of each factor type are linked through changes in λ. Moving southeast along the contour accompanies a rise in λ, which converges to one at the point where $w_b = \theta_b p$ and $w_a = 0$. As λ approaches zero, we reach the point where $w_b = 0$ and $w_a = \theta_a p$.

In Fig. 2, the zero profit isocost curve for the Y sector (RT) is superimposed

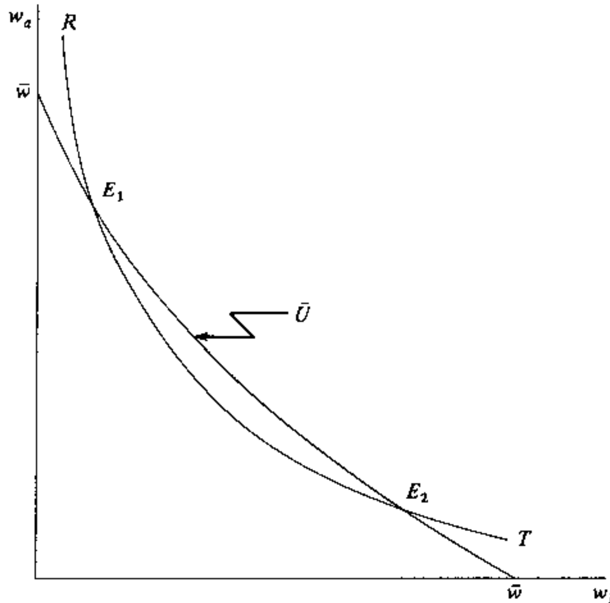

Fig. 2

on $\bar{U}$. The curve RT is the locus of (w_a, w_b) pairs that lead to zero profits in sector Y, therefore satisfying the last equilibrium condition. Since the unit cost function is concave in input prices, RT is convex to the origin. Moreover, since we have chosen good Y as the numeraire, changes in p do not affect RT. Finally, the absolute value of the slope of RT is equal to the factor intensity ratio in this sector (R_b/R_a).

We are now in a position to describe equilibrium. The curve RE_1E_2T corresponds to the factor price frontier, with E_1 and E_2 the only potential equilibria characterised by the production of both goods. For low values of p, U will lie everywhere below RT, and the only equilibrium will entail specialisation in good Y. This follows since the price of good X will be so low that firms in the Y sector will be able to bid away all X sector workers. As p rises, the two curves will eventually intersect as in Fig. 2, and each intersection will determine possible equilibrium values of w_a, w_b, λ, and θ_a. For any given (w_a/w_b), there is a unique factor intensity consistent with cost minimisation in sector Y (it can be derived from (2)). This relationship along with the steady state conditions in (13), and the factor market clearing conditions in (10) gives us five equations in five unknowns (M, R_a, R_b, S_a, S_b). Provided that there are strictly positive values of these variables that satisfy the equations, we have a diversified production equilibrium.

A simple geometric argument borrowed from international trade theory provides additional insight into the conditions under which a diversified production equilibrium exists. We begin by assuming that p is such that $\bar{U}$ and RT are just tangent. At this price level, there is only one sector Y wage vector and one value of λ potentially consistent with diversified production. For these wages, the unique value of (R_a/R_b) consistent with cost minimisation in sector Y can be derived from (2). In addition, given λ, the steady state conditions can be used to find the factor intensity of sector X (measured as $(M+S_a)/(M+S_b)$). These factor intensities are drawn in Fig. 3 under the

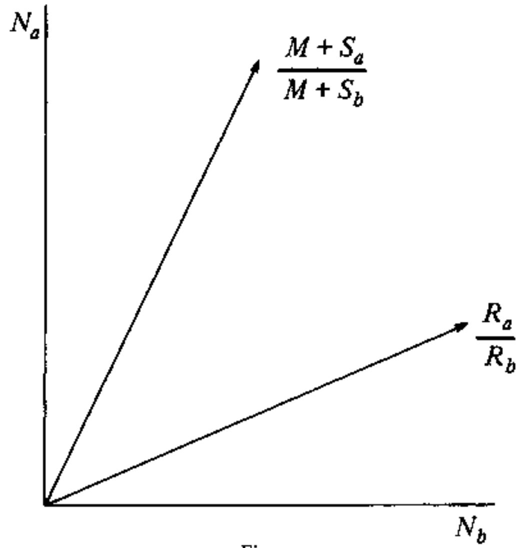

Fig. 3

assumption that sector X is relatively more A-intensive than sector Y. The area between the factor intensities is known as the diversification cone. If the economy's endowment lies in this cone, a diversified production equilibrium exists for this value of p.

As p rises, $\bar{U}$ and RT will intersect more than once for each price.[9] Increases in p cause both sectors to become more $B(A)$ intensive at intersections such as E_1 (E_2) in Fig. 2. This follows since increases in p are accompanied by increases (decreases) in (w_a/w_b) and decreases (increases) in λ at E_1 (E_2). The diversification cones for E_1 and E_2 therefore move in opposite directions as p rises (one rotates clockwise and the other counterclockwise). For each p, if the

[9] While the RT curve will always be convex, all that we can say about the $\bar{U}$ curve is that it is downward sloping and will eventually cut both axes. As such, there may be more than two intersections for any given p. However, this is not relevant for our purposes as we make clear in the text.

economy's endowment lies in neither diversification cone, no diversified production equilibrium exists; if it lies in only one cone, there is a unique diversified production equilibrium; and if it lies in the overlapping region of the cones, multiple equilibria will exist. However, since we are presently only interested in the normative properties of equilibria in this model and do not address issues of comparative statics, the existence of multiple equilibria is of no concern. The results that follow apply to all equilibria that are characterised by diversification of production. For a deeper analysis of equilibrium and the problems associated with multiple equilibria, see Davidson *et al.* (1986).

III. EFFICIENCY PROPERTIES OF EQUILIBRIA

A. Allocative Efficiency and the Production of Jobs

The partial equilibrium modelling of search processes indicates that the level of production is usually inefficient when search is required to find a job. The same result applies in our model as well. In any steady state equilibrium, there is an allocative inefficiency in the sense that too little X is produced relative to the production of good Y. In particular, we show that from an equilibrium where both goods are being produced, it is always possible to raise social welfare by increasing X sector production, *holding production techniques (i.e. factor intensities) constant.* From this standpoint alone the economy's equilibrium unemployment rate is actually too low, as the economy's unemployment rate is positively correlated with the relative size of sector X, *ceteris paribus.*

Intuitively, the inefficiency in the mix of outputs is due to an externality involved in moving to the X sector. When one additional person of type A moves to the X sector, the steady state number of matches increases. The individual mover can expect to earn V_{as}, which includes the expected income from her own possible matches. However, she ignores the increased expected income received by her partner because of the match.

We formally show this result by totally differentiating the social welfare function subject to λ, and therefore θ_a, held constant. We then show that this differential is positive when evaluated at equilibrium, and that social welfare can be increased by increasing the production of good X.

We measure the welfare of the economy by W, the summation of expected income where

$$W = M(1-\tau)^2(V_{am}+V_{bm}) + M[1-(1-\tau)^2](V_{as}+V_{bs}) + S_a V_{as} + S_b V_{bs} + R_a V_{ay} + R_b V_{by} \quad (15)$$

and where $M(1-\tau)^2$ is the number of individuals that *begin* each period matched and $S_i + M[1-(1-\tau)^2]$ is the number of type i searchers at the beginning of the period.[10]

We can use the production and steady state equations to express W as a

[10] $\tau(1-\tau)\,M$ previously matched workers must begin searching again since their partner has died. $(1-\tau)\,S_i$ searchers from the last period have survived, and $(M+S_i)\,\tau$ new workers entered the sector and began searching. These terms sum to $S_i+[1-(1-\tau)^2]\,M$ searchers.

function of R_a and R_b and increase the two variables in a manner that keeps λ, and thus θ_a, constant. The derivative of W can then be evaluated at the equilibrium to determine the impact of the experiment.[11]

To find the rate at which R_a must be substituted for R_b in order to keep λ constant, we can use the steady state conditions (13) to substitute S_a and S_b into one of the adding-up identities (10). We then use the remaining adding-up identity to substitute for M. Totally differentiating the remaining equation and setting $d\lambda = 0$ we find

$$dR_b = \left(\frac{M+S_b}{M+S_a}\right) dR_a. \tag{16}$$

THEOREM 1. *Increasing the size of the search sector while holding the mixture of searchers fixed increases welfare and increases unemployment.*

Proof. First, we totally differentiate (15) with respect to R_a and R_b

$$dW = \frac{\partial W}{\partial R_a} dR_a + \frac{\partial W}{\partial R_b} dR_b.$$

Substituting from (16)

$$dW = \left(\frac{\partial W}{\partial R_a} + \frac{\partial W}{\partial R_b} \frac{M+S_b}{M+S_a}\right) dR_a;$$

or

$$dW = \left[V_{ay} - V_{as} + \frac{M+S_b}{M+S_a}(V_{by} - V_{bs}) - \frac{M(1-\tau)^2}{M+S_a}(V_{aM} - V_{as} + V_{bM} - V_{bs})\right] dR_a < 0.$$

The terms $(V_{is} - V_{iy})$ vanish in equilibrium from (14), and the remainder of the expression is strictly negative. Thus, it is always possible to reduce R_a and R_b in the proportions described in (16) and increase welfare. Q.E.D.

Theorem 1 applies in any equilibrium, regardless of whether the economy is on the production possibilities frontier (an issue that is addressed below). However, the theorem does not say that a subsidy to sector X is necessarily Pareto improving. The sector must be expanded in such a way as to keep λ constant. Whether moving to the constrained Pareto optimum entails an ultimately larger quantity of good X depends upon the interaction of output changes and changes in λ. We consider this question in Section IV.

Theorem 1 also establishes the existence of an externality that is distinct from the congestion externalities found in similar search models. Since the search sector is expanding in a manner that keeps λ constant, the new searchers do not

[11] Note that (15) calculates social welfare *ex ante*, that is, before the completion of the matching process. Alternatively, we could choose to express W *ex post*, in which case we would use $\tilde{W}$, where

$$\tilde{W} = M(V_{am} + V_{bm}) + \frac{1-\pi_a}{\pi_a} M[1-(1-\tau)^2] V_{as} + \frac{1-\pi_b}{\pi_b} M[1-(1-\tau)^2] V_{bs} + R_a V_{ay} + R_b V_{by}.$$

Theorem 1 does not depend upon the choice of *ex ante* or *ex post* welfare.

impose any congestion externalities on others in the sector.[12] There is a way to interpret the externality exposed by Theorem 1 that sheds some light on an important issue that emerges in our analysis that has not been emphasised in prior literature. Many economists would argue that the value of a job is fully reflected in the output that it produces. In our model, this is not true. To see this, we substitute from (3), (6) and (7) into (15) in order to express the sum of expected income as

$$W = \frac{(pX+Y)}{\tau} + M(1-\tau)^2(V_{aM} - V_{as} + V_{bM} - V_{bs}). \tag{17}$$

The first term is the market value of output discounted by the death rate, and the second is the total value of the matches in the X sector. There are $M(1-\tau)^2$ individuals of type A matched at the beginning of each period, each enjoying a bonus income of $(V_{am} - V_{as})$. There is also an analogous term for the B workers who are matched. Matches or jobs can therefore be seen as a joint output of the X sector, for additional output can only be produced by producing extra jobs. These jobs have welfare consequences in that their holders experience extra expected lifetime income.

The economy will choose the wrong mix of outputs because it undervalues the matches inherently created by search sector activity. Their value emerges as a result of the increase in the conditional probability of future employment. The possibility that jobs have inherent economic value raises a large number of policy questions. This is especially true if, as we show below, the value of jobs may vary with the unemployment rate; or if some jobs have more value than others (in the present context, Y sector jobs have no value).[13]

[12] As pointed out by an anonymous referee, this result is analogous to results that have been obtained in the matching and marriage literatures. In any two sided matching model, there are two sources of inefficiency. First, agents ignore the fact that changes in their own behaviour affect the probabilities of successful search for others. This type of externality is commonly referred to as a congestion externality. Second, agents ignore the capital loss imposed on potential future partners when deciding whether or not to 'marry' (create a partnership). In the marriage literature, it is well known that in the absence of congestion externalities, agents are too prone to get married due to the second type of externality. This follows since agents ignore the capital loss their marriage would impose on all potential future mates. In our model, these externalities apply to the decision of whether or not to enter the sector and this externality leads to a search sector that is too small. Theorem 1 establishes the existence of this externality and its consequences, holding any congestion externalities fixed (by holding λ fixed).

[13] We should point out that choosing the sum of utilities as the objective function is crucial for the results. In making this choice, we are implicitly assuming the absence of lump sum transfers and pursuing a constrained Pareto optimum. That is, we are asking what the optimal allocation of resources is, given that output is distributed according to marginal productivity in the agricultural sector and according to the bargaining outcome in the urban sector. Alternatively, if costless lump sum transfers were available, we would write the social welfare function as $W^* = (N_a+N_b)\,U[X/(N_a+N_b), Y/(N_a+N_b)]$. Here we specify the objective function to be maximised as the sum of the direct utility functions when output is equally distributed. Maximising this expression with respect to λ gives $\Omega = 0$. That is, productive efficiency is, of course, necessary. The first order condition with respect to M gives

$$U_x + U_Y \left\{ Y_a \left[\frac{1-(1-\tau)^2(1-\pi_a)}{\pi_a} \right] + Y_b \left[\frac{1-(1-\tau)^2(1-\pi_b)}{\pi_b} \right] \right\} = 0.$$

Evaluating this expression in equilibrium gives the following:

$$U_x + U_Y \left[Y_a \frac{\theta_a p}{w_a} + Y_b \frac{(1-\theta_a)\,p}{w_b} \right] = U_x + U_Y\, p = 0.$$

B. Production Efficiency and Unemployment

Because the existing literature on search only allows for a single production technique, the question of whether the economically efficient production technique will be employed in equilibrium has never been asked. As we presently show, the answer to this question is almost always negative. The intuition behind this result is an extension of the externalities argument used to show that sector X is too small in equilibrium. Recall, type i workers in the search sector ignore the fact that each match they are part of leads to an increase in their partner's expected income. This positive externality is what makes the search sector too small. In addition, unless the search sector is perfectly symmetric ($\lambda = \frac{1}{2}$), the externalities ignored by the agents will not be of equal size. Since a match is more valuable to a worker in the majority (due to the fact that they have a more difficult time finding a partner), the externality ignored by those in the minority is larger. This immediately leads to the result that relatively too few of the minority type enter the search sector. In general, the search sector is too asymmetric in equilibrium. Increasing the symmetry of the sector enhances productive efficiency while simultaneously reducing the sector specific unemployment rate through an increase in the production of matches.

Our analysis proceeds in three steps. First, we derive the conditions for production efficiency. This involves characterising the production function for sector X as a whole and equating marginal rates of technical substitution across sectors. In step two, we specify the relationship between the equilibrium value of λ (the proportion of searching workers of type A) and the value of λ needed for production efficiency. Finally, we show that a local movement towards productive efficiency is possible and that it must reduce unemployment.

Using the steady state conditions in (13) and the identities in (10) to substitute for R_a, R_b, S_a and S_b allows us to write Y as a function of M and λ,

$$Y\left(N_a - M\left\{\frac{1-(1-\tau)^2[1-\pi_a(\lambda)]}{\pi_a(\lambda)}\right\},\quad N_b - M\left\{\frac{1-(1-\tau)^2[1-\pi_b(\lambda)]}{\pi_b(\lambda)}\right\}\right).$$

The necessary condition for steady state production efficiency is found by choosing λ to maximise Y holding M constant. The first order condition for this maximisation problem reduces to

$$\Omega = \frac{Y_a}{Y_b} + \left[\frac{\pi_a(\lambda)}{\pi_b(\lambda)}\right]^2 \frac{\pi_b'(\lambda)}{\pi_a'(\lambda)} = 0.$$

LEMMA 1. *$\Omega = 0$ is a necessary condition for production efficiency.*

To gain some insight into Lemma 1, we now examine production in the

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This is just the condition for utility maximisation for consumers. The implication is that there is no externality in search sector production when lump sum transfers are possible. The externality stems from the higher expected lifetime income of a matched worker. With lump sum transfers, this surplus can be distributed among less fortunate individuals; therefore there is no reason to expand the X sector.

search sector in greater detail. Let $Q_i = M + S_i$ denote the total number of type i workers in sector X. Then, for any given λ, (13) tells us that to produce X units of the search sector good we must have

$$Q_i = \frac{1-(1-\tau)^2[1-\pi_i(\lambda)]}{\pi_i(\lambda)} X. \tag{18}$$

By varying λ, one can obtain all the combinations of Q_a and Q_b that lead to the production of exactly X units of the search sector good. That is, one can obtain the search sector isoquant. By solving the Q_a equation for λ, substituting into the Q_b expression, and totally differentiating, we can obtain the slope of this isoquant.[14] Straight forward calculations yield:

$$\frac{dQ_b}{dQ_a} = \left[\frac{\pi_a(\lambda)}{\pi_b(\lambda)}\right]^2 \frac{\pi_b'(\lambda)}{\pi_a'(\lambda)}.$$

To guarantee that the isoquants are convex, we assume $\pi_i''(\lambda) \leqslant 0$ so that the above expression is decreasing (in absolute value) in λ.[15] With this assumption, $\Omega = 0$ is also a sufficient condition for production efficiency.

By now it should be clear that Lemma 1 has a natural interpretation; for production efficiency λ should be chosen to equalise the marginal rates of technical substitution across sectors. If $\Omega > 0$, then the marginal rate of technical substitution is too high in sector Y. By substituting type A workers for type B workers in the Y sector in a manner that keeps us on the X isoquant, we can increase the production of good Y without reducing the production of good X. In other words, the Y sector is too B intensive. A similar argument can be used to show that if $\Omega < 0$, production efficiency can be enhanced by reducing the A intensity of the Y sector.

We now turn to the question of whether this condition for production efficiency is met in equilibrium. By cost minimisation in sector Y (or, equation (2)), in equilibrium we must have $(Y_a/Y_b) = (w_a/w_b)$. We therefore define

$$\Omega^e = \frac{w_a}{w_b} + \left[\frac{\pi_a(\lambda^e)}{\pi_b(\lambda^e)}\right]^2 \frac{\pi_b'(\lambda^e)}{\pi_a'(\lambda^e)},$$

where the superscript e denotes equilibrium values. Clearly, we have productive efficiency if and only if $\Omega^e = 0$.

The value of θ_a is given in (9). Combining this with (3), (6), and (14) yields w_a and w_b. We obtain

$$\frac{w_a}{w_b} = \frac{\pi_a(\lambda^e)}{\pi_b(\lambda^e)}$$

which leads to our next result.

LEMMA 2. *$\Omega^e \gtreqless 0$ as $\lambda^e \lesseqgtr \frac{1}{2}$.*

Proof. By symmetry, when $\lambda^e = \frac{1}{2}$, $\pi_a = \pi_b$ and $\pi_a' = -\pi_b'$ so that $\Omega^e = 0$.

[14] Equation (18) implicitly defines a production function $X = X(Q_a, Q_b)$ for the sector as a whole, where the inputs are the total populations of each type living in the sector.

[15] This is actually a stronger assumption than we need. All that is required is that $(\pi_a/\pi_b)(\pi_b'/\pi_a')$ is decreasing in λ.

The rest of the proof follows trivially from our assumption that $\pi_i''(\lambda) \leqslant 0$. Q.E.D.

We have demonstrated that the economy operates on the production possibilities frontier if and only if the equilibrium value of λ is one half. Therefore, the economy is almost always operating in a productively inefficient manner.

We now turn to the issue of unemployment. In every period, after the matching process has occurred, some workers of each type find themselves unemployed. Let μ denote the unemployment rate. Then from (13) we have

$$\mu = \frac{S_a + S_b}{N_a + N_b} = \frac{M[1-(1-r)^2]}{N_a + N_b}\left[\frac{1-\pi_a(\lambda)}{\pi_a(\lambda)} + \frac{1-\pi_b(\lambda)}{\pi_b(\lambda)}\right]. \qquad (19)$$

Lemma 3 specifies the relationship between μ and λ, assuming that the output of sector X remains fixed.

LEMMA 3. $$\left.\frac{\partial \mu}{\partial \lambda}\right|_{X=M=\text{constant}} \gtreqless 0 \text{ as } \lambda^e \gtreqless \tfrac{1}{2}.$$

To prove the lemma, simply differentiate (19) with respect to λ, holding M constant, and use the assumption that $\pi_i''(\lambda) \leqslant 0$. This result indicates that if the equilibrium level of λ does not equal one half, then any local movement towards one half will reduce unemployment. We are now in a position to prove the main result of this sub-section.

THEOREM 2. *Increasing the symmetry of the search sector population, while holding the level of that sector's output constant, improves productive efficiency and reduces unemployment.*

Proof. From Lemma 1, we know that $\Omega = 0$ is necessary for productive efficiency, and from Lemma 2, this can only occur in equilibrium if $\lambda = \frac{1}{2}$. Otherwise, $\lambda^e < \frac{1}{2}$ implies that $\Omega^e < 0$ and the marginal rates of technical substitution are not equal. Substitution of type A workers for type B workers in the search sector, holding X constant, increases λ and from Lemma 3, reduces unemployment. Similarly, $\lambda^e > \frac{1}{2}$ implies that $\Omega^e > 0$ and substitution of type B for type A workers in the search sector can simultaneously keep X constant, increase Y, and reduce unemployment. Q.E.D.

The theorem *does not say that the optimal λ is one half.* Rather it says that from an equilibrium in which λ does not equal one half, it is possible to make a *local* movement towards one half which improves productive efficiency and reduces unemployment. Another way of looking at this result is that production in the search sector in equilibrium is generally too asymmetric; it makes inefficient use of the relatively abundant type of worker.

Theorem 2 also establishes the existence of an externality distinct from the externality exposed by Theorem 1.[16] By changing λ, holding the size of the

[16] Returning to the discussion in note 12 on p. 868, Theorem 2 isolates the role of the congestion externality, holding the size of the search sector fixed. For a summary of the results in the marriage literature see Mortensen (1986*b*).

search sector fixed, the matching probabilities change. It is well known that in two sided matching models, such congestion externalities may have ambiguous effects (see for example, Pissarides (1984*b*) or Mortensen (1986*b*)). We have demonstrated that for a specific class of search technologies these congestion externalities lead to a search sector that is too asymmetric.

IV. PROPERTIES OF THE SOCIAL OPTIMUM

A. Equilibrium and Optimal Unemployment

It is natural to think of an economic planner for this economy as having two decisions to make: to choose the factor mix in each sector such that production is along the production possibilities frontier and to choose the mix of output such that welfare is maximised. In Section III *B*, we showed that the competitive equilibrium of this economy fails in the first task. In a sense, the unemployment associated with the equilibrium mix of inputs is too high in that unemployment falls as we move from the market frontier to the production possibilities frontier *keeping output of the X sector constant.*

On the other hand, in Section III *A*, we established that a movement from the equilibrium in the direction of more search sector production *keeping the factor mix in that sector constant* will improve welfare and concommitantly increase unemployment.

Is unemployment too high? From the above discussion, one is tempted to say that unemployment is too high for productive efficiency (λ is too far from one half in equilibrium) and too low for allocative efficiency (M is too small). However, one needs to be more careful. The correct way to formulate the question is to compare the equilibrium and optimal levels of unemployed workers. To do so, we maximise W with respect to M and λ. In addition, we simplify the analysis by concentrating on a particular specification of the search technology.

The search technology that we focus on here has probability functions of the form $\pi_a(\lambda) = 1-\lambda$ and $\pi_b(\lambda) = \lambda$. These probability functions would be generated by a search process in which all searchers are randomly paired at the beginning of each period.[17]

Using this specification of $\pi_i(\lambda)$, (3), (6), and (9) we can rewrite W as

$$W = \frac{1}{\tau}(pM+Y) + M(1-\tau)^2 \frac{p}{2-(1-\tau)^2}. \tag{20}$$

The optimal value of M and λ are the ones that maximise (20). The first order conditions are as follows:

$$\frac{\partial W}{\partial M} = \frac{1}{\tau}\left[p - Y_a \frac{1-\lambda(1-\tau)^2}{1-\lambda} - Y_b \frac{1-(1-\lambda)(1-\tau)^2}{\lambda}\right] + \frac{p(1-\tau)^2}{2-(1-\tau)^2} = 0, \tag{21}$$

$$\frac{\partial W}{\partial \lambda} = \frac{[1-(1-\tau)^2]}{\lambda^2} Y_b \Omega = 0. \tag{22}$$

[17] The underlying matching technology that generates this probability function is
$N = \min[S'_a, S'_b, (S'_a S'_b)/(S'_a + S'_b)^2]$.

From (22), $\Omega = 0$ is necessary for the maximisation of (20). However, this results from the fact that the value of an additional job, the last term in (20), is independent of λ, a point discussed in detail below.

Evaluating (21) and (22) in equilibrium, we have

$$\frac{\partial W}{\partial M} = \frac{p(1-\tau)^2}{2-(1-\tau)^2} > 0, \tag{23}$$

$$\frac{\partial W}{\partial \lambda} = -\Omega^e \left\{ \frac{[1-(1-\tau)^2]\, Y_b}{(\lambda^e)^2} \right\} \lesseqgtr 0 \quad \text{as} \quad \lambda^e \gtreqless \tfrac{1}{2}.$$

One cannot conclude from (23) that the Pareto optimal M exceeds the equilibrium M, or from (24) that the Pareto optimal λ is nearer to one half than the equilibrium λ. For example, adjusting M towards its optimal value causes the left hand side of (22) to change. These cross effects may be strong enough to produce anomalous results.[18] However, even if we could conclude that equilibrium M is too low and λ^e is too far from one half, we would not be able to say much about unemployment. As we have seen in the previous sections, increasing M increases unemployment, while adjusting λ towards one half reduces unemployment. Equilibrium unemployment may therefore be either too high or too low.

Some insights can be provided by examining a few special cases in which definite conclusions can be drawn. For example, suppose that $\lambda^e = \frac{1}{2}$. In this case, the economy operates along the production possibilities frontier. The economy ignores the extra value of jobs, however, and therefore it is not at the Pareto optimum, which requires a movement towards increasing production in the search sector. As this sector grows in size, unemployment increases. In addition, the adjustment in good X moves λ away from one half. This movement causes unemployment to rise even further, since the matching technology is used less efficiently. We can therefore conclude that in this case equilibrium unemployment is too low.

On the other hand, suppose that there is no value to an additional job. This would be true if the death rate were one hundred percent ($\tau = 1$) or if the government could institute costless lump sum transfer from employed individuals to unemployed individuals (see note 13 on p. 868). In this case, movement towards the production possibilities frontier improves matters and reduces unemployment *for the given* X. However, this value of X may not be optimal at the frontier. As long as the required adjustment in X is downward, we can say that unemployment in equilibrium is too high.

B. The Search Technology and the Pareto Optimality of Production Efficiency

While the competitive equilibrium will generally be productively inefficient, it may also be true that productive efficiency is not characteristic of the Pareto

[18] This issue relates to an old controversy in environmental economics concerning whether activities which generate external costs are too numerous in equilibrium. The controversy is generally regarded to have been settled by Diamond and Mirlees (1973), who derived conditions under which one can unambiguously rank the magnitudes of equilibrium and optimal values of activities which generate external effects. In our model, these conditions reduce to the relative A or B intensity of the search sector.

optimum. Recall that we can write the welfare function as the sum of two terms: the value of output and the value of jobs

$$W = \frac{1}{\tau}(pX + Y) + M(1-\tau)^2 Z \tag{25}$$

where $Z = (V_{am} - V_{as} + V_{bm} - V_{bs})$ denotes the value of a match. Substituting from (6), (7), and (9) yields

$$Z = \frac{p[2 - \pi_a(\lambda) - \pi_b(\lambda)]}{2 - (1-\tau)^2[2 - \pi_a(\lambda) - \pi_b(\lambda)]}. \tag{26}$$

In (26), Z generally depends on λ. The exception occurs when $\pi_a(\lambda)$ and $\pi_b(\lambda)$ sum to a constant and Z becomes independent of λ. With virtually any other search technology, Z will remain dependent on λ. To see why this is important, differentiate W with respect to λ to obtain:

$$\frac{\partial W}{\partial \lambda} = \frac{1}{\tau}\frac{\partial(pX+Y)}{\partial \lambda} + M(1-\tau)^2 \frac{\partial Z}{\partial \lambda}. \tag{27}$$

The first term is zero if and only if production efficiency is attained ($\Omega = 0$). If π_i is a linear function of λ, the second term is always zero and therefore, production efficiency is necessary in order to maximise welfare. However, with any non-linear probability function $\partial Z/\partial \lambda$ would be non-zero, and there is no reason to expect it to equal zero when $\Omega = 0$. Production efficiency would no longer be necessary for welfare maximisation. The reasoning is as follows. There is a tension between the value of λ which minimises the opportunity cost of X sector production (measured in terms of foregone production of good Y), which is production efficiency, and the value of λ which maximises the value of jobs. One can show that while production efficiency requires λ to be closer to one half than its equilibrium value, the maximisation of the value of a job requires λ to be farther from one half. In other words, jobs are more valuable when there is more unemployment. Thus, in this case, the welfare function achieves its maximum value at a point inside the production possibilities frontier. There is a bliss point.

V. CONCLUSION

In the context of a simple general equilibrium search model, we addressed the issue of the relative levels of equilibrium and optimal unemployment. Unlike the modelling that had previously been undertaken, the two sector, two factor general equilibrium model allowed us to isolate two influences on the unemployment rate: the factor mix, which determines unemployment holding search sector output constant; and the size of that sector. We demonstrated that unemployment is too high in the sense that a given amount of the good produced by the sector with unemployment can be produced with a lower opportunity cost *and* with lower unemployment. We also demonstrated that unemployment is too low in the sense that holding the factor mix constant, both welfare and unemployment increase as the sector with unemployment increases. The reason is that in a search economy jobs have an inherent value which is not fully reflected in current production. A job enhances both current production and the conditional probability that future production will occur. The latter following from the fact that the probability of holding a job in the future is higher for those currently employed. When output is produced in the search sector, the economy simultaneously produces jobs. Steady state welfare not only includes the value of output, but also the value of these jobs.

Finally, we note that once jobs themselves are a recognised element of welfare, the spectre of optimal production *in*efficiency is raised. There may be a tension between the role of the factor mix in minimising the opportunity cost of production and its role in maximising the value of jobs. We addressed this issue and showed that there is some comfort in one particular case; if the search technology leads to a linear probability function, production efficiency remains a necessary condition for Pareto optimality.

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References

Azariadis, C. (1975). 'Implicit contracts and underemployment equilibrium.' *Journal of Political Economy* vol. 83, pp. 1183–202.

Baily, M. (1974). 'Wages and employment under uncertain demand.' *Review of Economic Studies* vol. 41, pp. 37–50.

Binmore, K. (1982). 'Perfect equilibria in bargaining models.' London School of Economics Paper No. 8258.

Davidson, C., Martin, L. and Matusz, S. (1986). 'The structure of simple general equilibrium models with frictional unemployment.' Econometrics and Economic Theory Paper No. 8604, Michigan State University.

Diamond, P. (1981). 'Mobility costs, frictional unemployment, and efficiency.' *Journal of Political Economy* vol. 89, pp. 798–812.

—— (1982). 'Wage determination and efficiency in search equilibrium.' *Review of Economic Studies* vol. 49, pp. 217–28.

—— (1984). *A Search Equilibrium Approach to the Micro Foundations of Macroeconomics: The Wisksell Lectures.* Cambridge, Mass: MIT Press.

—— and Mirlees, J. (1973). 'Aggregate production with consumption externalities.' *Quarterly Journal of Economics* vol. 87, pp. 1–24.

Hosios, A. (1986). 'On the efficiency of matching and related models of search and unemployment.' University of Toronto working paper.

Jovanovic, B. (1984). 'Matching, turnover, and unemployment.' *Journal of Political Economy* vol. 92, pp. 108–22.

McLennan, A. (1982). 'A noncooperative definition of bargaining.' University of Toronto Working Paper No. 8303.

Mortensen, A. (1982*a*). 'The matching process as a non-cooperative/bargaining game' In *The Economics of Information and Uncertainty* (ed. J. J. McCall), Chicago: University of Chicago Press.

—— (1982*b*). 'Property rights and efficiency in mating, racing, and related games.' *American Economic Review* vol. 72, pp. 968–79.

—— (1986*a*). 'Job search and labor market analysis.' In *The Handbook of Labor Economics*, vol. II, (ed. O. Ashenfelter and R. Layard), Amsterdam: Elseviers Science Publishers.

—— (1986*b*). 'Matching: Finding a partner for life or otherwise.' Northwestern University working paper.

Nickell, S. J. (1979). 'Estimating the probability of leaving unemployment.' Econometrica vol. 47, pp. 1249–66.

Pissarides, C. A. (1984*a*). 'Efficient job rejection.' Economic Journal vol. 94, *Supplement*; pp. 97–108.
—— (1984*b*). 'Search intensity, job advertising, and efficiency.' *Journal of Labor Economics* vol. 2, pp. 128–43.
Rubinstein, A. (1982). 'Perfect equilibrium in a bargaining model.' *Econometrica* vol. 50, pp. 97–109.
—— and Wolinsky, A. (1985). 'Equilibrium in a market with sequential bargaining.' *Econometrica* vol. 53, pp. 1133–50.