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Equilibrium in Servicing Industries: An Economic Application of Queuing Theory*

The purpose of this article is to investigate the nature of equilibrium in markets in which service and waiting time play an important role. I show that if consumers do not know which firms are charging which prices, all firms charge the same price. If firms reveal their prices by advertising, the market separates with consumers with a high (low) cost of time buying from firms with high (low) prices and short (long) queues. If firms are allowed to advertise, they will, but they benefit from a collusive agreement restricting advertisement provided the agreement is enforceable.

I. Introduction

This article explores the nature of equilibrium in servicing industries. Servicing industries differ from the standard industries in that they require consumers to spend a nontrivial amount of time in obtaining the product. Thus, purchasers are interested in more than just the money price of the good; they evaluate the cost of the good based on its full price, which is the money price plus the cost of time that is needed to purchase the product. Typical industries in which waiting time and service time play an important role are the taxi industry, supermarkets, restaurants, full-service gas stations, and the market for medical services.

The primary goal of this article is to investigate how well the market accommodates consumers with different time preferences in such industries. Particular attention will be paid to the way in which the answer to this question depends on the amount of information consumers are assumed to possess. For example, if consumers do not know which firms are charging which money prices, will we observe different types of firms in

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equilibrium, or will all firms charge the same price? I will also be interested in determining which information structure is most likely to be observed in equilibrium. That is, if consumers do not possess complete price information, is it in the firms' interest to reveal their price by advertising?

In spite of the obvious importance of servicing industries, the theoretical analysis of firm and consumer behavior in this setting has been quite limited. Mann (1974) developed a very simple model of servicing industries and showed that if consumers vary in their valuation of time, equilibrium will be characterized by both high-priced firms with short queues and low-priced firms with long queues. However, in her paper, consumer behavior is not modeled and, in fact, is not optimal. Hence, at best she provided only a "partial" partial analysis of such industries. DeVany (1976) and DeVany and Saving (1977) considered the competitive and monopolistic solutions in servicing markets, but in both of their models all consumers were assumed to have the same cost of time. Since it is clear that consumers do place different values on time, any model of such industries should take this into account. In addition, in all three papers mentioned above, consumers were assumed to possess complete price information. No one has examined how these industries function when consumers face incomplete information.

This article divides into two sections. In the first section it is assumed that consumers know the distribution of prices in the market but do not know which firms are charging which prices. Since consumers must choose firms randomly in this case, the firm's arrival rate must be independent of the price it charges, and therefore, since there is no advantage to charging a lower price than the other firms, equilibrium is characterized by all firms charging the same price. In the second section it is assumed that consumers have complete price information but cannot know the queue length at any given firm without traveling to the firm and actually observing the queue. Thus, even though consumers possess complete price information, an element of search is still involved due to the inherent uncertainty that must exist in servicing industries.[1] Since service times and arrival rates are random, it is virtually impossible for the consumer to know the *full* price of the good before arriving at the firm.[2] The major distinction between the two models is that, with complete price information, consumers always know the "type" of firm they are going to, and hence the firms' arrival rates are a function of the price they charge. This fact benefits consumers since the market is able to separate with consumers with a high cost

1. In some industries, the uncertainty can be reduced by allowing consumers to make appointments to be served. In Davidson (1982) it is shown that allowing appointments does not change the qualitative results derived in the article.

2. Even when appointments can be made, the uncertainty of service times usually results in queue formation, and hence the full price cannot be known before search.

of time paying higher prices in order to avoid the longer wait at the more crowded, lower-priced stores.

Finally, this article also provides an explanation of how prices can differ in equilibrium even though they are publicly known. Since waiting lines are known to be longer at stores with lower prices, consumers with higher values of time frequent the more expensive stores. Thus, consumers sort themselves according to the different expected waiting times at different stores. Of course, this principle is more general; any congestion externality will do so long as purchasers differ in the disutility the externality imposes on them. The queuing model presented in this paper is the best example of such an externality since it yields comparatively tractable results.[3]

II. Incomplete Information

Consumer Behavior

In this section it is assumed that the consumers know the distribution of prices in the market, $F(p)$, but do not know which firms are charging which prices. They therefore must search for the lowest-priced firm. I assume that all consumers face a common monetary cost of search denoted by b with $b > \delta$ for some $\delta > 0$ (so that b is bounded away from zero). Service time at each firm is assumed to be exponentially distributed with mean $1/\mu$, and, finally, I assume that the number of firms is large enough that the queue lengths across firms may be considered independent.[4]

When the distribution of prices is known, the consumer's optimal search strategy is to search until she finds a full price below some predetermined reservation price.[5] This reservation price is the value that, when using this rule, minimizes the expected cost of obtaining the product. Consumers search by choosing a firm at random and, on arrival, observing the queue length and price. If the price plus the

3. There is, of course, a large literature on quality choice by producers (e.g., Shapiro 1982, 1983; or Riordan 1986). Since expected service time is a characteristic of a good (and sometimes used as a measure of quality), these papers are, in a sense, related to this study. However, these papers deal almost exclusively with the case in which prices are known (quality is generally assumed to be unknown). Therefore, consumers use prices as a signal for quality and an equilibrium results. In this article, I investigate how the type and level of information that consumers possess (concerning price) affect the equilibrium outcome.

4. In reality, the seller can, at a cost, alter the distribution of service times. However, allowing it to be chosen by the firms greatly complicates the analysis. Moreover, before studying complex models it is essential to understand the forces at work in simpler (and often unrealistic) models. Therefore, the model developed in this article represents only a first attempt at understanding how servicing industries cope with heterogeneous consumers and incomplete information.

5. See Rothschild (1973) for the result in standard models, and for an extension to queuing models, see Glazer and Hassin (1983).

waiting time is less than their reservation price, they join the queue. If the sum is greater than their reservation price, they continue to search. Throughout the article I assume that service is on a first come–first served basis, so that if there are n people in the queue when the consumer arrives, expected waiting time is $(n + 1)/\mu$. If we let t_j denote the per-unit cost of time for consumer j, the expected full price of the good is given by $p + [(n + 1)/\mu]t_j$. For simplicity it is assumed that all consumers have the same demand curve given by

$$D(B) = \begin{cases} 1 & \text{if } p + \left(\frac{n+1}{\mu}\right) t_j \leq \hat{B}, \\ 0 & \text{otherwise,} \end{cases} \tag{2.1}$$

where B represents the full price and $\hat{B}$ is the consumer's choke price.

In order to solve the consumer's problem, we need to determine the expected cost of obtaining the good when B_j^* is the reservation price. If the consumer balks (chooses not to join the queue), he must pay b dollars to search again, and it takes w units of time to get to the next firm. Hence, the full cost of searching once is $b + wt_j$. If we let $G(B)$ denote the distribution of full prices, then the expected cost of obtaining the product when using reservation price B_j^* is

$$E(\text{cost}) = (b + wt_j)\sum_{n=1}^{\infty} n[1 - G(B_j^*)]^{n-1}G(B_j^*) + \frac{\int_0^{B_j^*} B\,dG(B)}{\int_0^{B_j^*} dG(B)}. \tag{2.2}$$

Quantity $[1 - G(B_j^*)]^{n-1}G(B_j^*)$ is the probability that the search lasts exactly n periods so that the first term in (2.2) gives the expected cost of search. The second term is the expected price the consumer will pay when buying only if $B \leq B_j^*$ (i.e., $E(B|B \leq B_j^*)$). Choosing B_j^* to minimize (2.2) yields the following first-order condition:

$$b + wt_j = \int_0^{B_j^*} (B_j^* - B)\,dG(B).$$

Writing this condition in terms of $F(p)$, the distribution of money prices, yields[6]

6. The consumer searches again if and only if the new full price is below B_j^*. If the queue at the new firm is of length x, this implies p, the new price, must satisfy $p + [(x + 1)/\mu]t_j \leq B_j^*$ or $p \leq B_j^* - [(x + 1)/\mu]t_j$. Hence, the upper bound of integration. The upper bound for the summation comes from noticing that if the new price is zero the consumer will join the new queue if and only if $[(x + 1)/\mu]t_j \leq B_j^*$ or $x \leq B_j^*(\mu/t_j) - 1$. Note that (2.3) can also be derived by simply equating the expected gain from one more search with the expected cost, assuming perfect and costless recall. This derivation is presented in Davidson (1982).

$$b + wt_j = \sum_{x=0}^{\left[B_j^*\left(\frac{\mu}{t_j}\right)-1\right]} \int_{p=0}^{B_j^*-\left(\frac{x+1}{\mu}\right)t_j} \left[B_j^* - p - \left(\frac{x+1}{\mu}\right)t_j\right] q(x|p)\, dF(p), \quad (2.3)$$

$$\text{subject to } B_j^* \leq \hat{B},$$

where $q(x|p)$ denotes the steady-state probability that x people are in line at any point in time at a firm charging price p.

Thus, to summarize, consumer j visits firm i and joins the queue if and only if $p_i + [(n_i + 1)/\mu)t_j \leq B_j^*$.

Firm Behavior

For simplicity I assume that there are only two types of consumers: those who value time and those who do not. Let α denote the steady-state proportion of consumers for which $t_j = 0$, with B_0^* their effective reservation price, and B_1^* the effective reservation price of those who value time. For this latter group I set $t_j = 1$. The assumption that $t_j = 0$ for some consumers is not necessary, but it greatly simplifies the analysis.[7]

In order to determine the firm's profit function, I first solve for $q(x|p)$. I assume that customers arrive at the firm according to a Poisson process with parameter λ, which depends on the ratio of the number of firms to the number of consumers, the rate at which consumers enter the market (the birth rate), and the rate at which consumers leave the market (the death rate). I discuss the determination of the equilibrium value of λ below, but, from the firm's point of view, λ is exogenous and independent of its price (since consumers do not know which firms are charging which prices). Note that the expected number of customer arrivals with $t_j = 0$ in a unit of time is $\alpha\lambda$.

I have assumed that the service rate is $1/\mu$, and thus I can define $z = \lambda/\mu$ as the "traffic intensity." Throughout this article I assume $z < 1$ in order to guarantee the existence of a steady-state solution (a standard result from queuing theory).

In order to determine $q(x|p)$, note that all consumers who do not value time join the queue (no matter how long) at any firm charging a price below B_0^*. In addition, for any given B_1^* and p_i, there is a critical queue length $n_i^* = [(B_1^* - p_i)\mu]$ such that consumers who value time join the queue at firm i if and only if the actual queue length is less than n_i^*. Then, for a steady-state solution we must have,[8] for $p_i \leq B_0^*$,

$$q(x|p_i) = zq(x-1|p_i) \qquad \text{for all } x \leq n_i^*, \qquad (2.4)$$

7. As stated, the assumption that a class of consumers places no value on time is not essential to the analysis but greatly simplifies several key expressions. All results would continue to hold in a model with two types of consumers, each valuing time, but at different positive values.

8. See, e.g., Gross and Harris (1974), pp. 40–46, or Haight (1957).

and

$$q(x|p_i) = \alpha z q(x - 1|p_i) \quad \text{for all } x > n_i^*, \tag{2.5}$$

$$\sum_{x=0}^{\infty} q(x|p_i) = 1. \tag{2.6}$$

Solving for $q(x|p_i)$, we obtain

$$q(x|p_i) = \frac{z^x(1 - z)(1 - \alpha z)}{(1 - z^{n_i^*+1})(1 - \alpha z) + z^{n_i^*+1}(1 - z)\alpha}, \quad \text{for all } x \leq n_i^*. \tag{2.7}$$

A customer balks (chooses not to join the queue) if and only if he values time *and* the queue consists of n_i^* or more people. Thus, the probability that any given customer balks is

$$\beta(p_i|p_i \leq B_0^*) \equiv (1 - \alpha)[1 - \sum_{x=0}^{n_i^*-1} q(x|p_i)],$$

or, from (2.7):

$$\beta(p_i|p_i \leq B_0^*) = \frac{(1 - \alpha)z^{n_i^*}(1 - z)}{1 - \alpha z - (1 - \alpha)z^{n_i^*+1}}. \tag{2.8}$$

Value $\beta(p_i)$ can be interpreted as the long-run proportion of consumers who choose not to join the queue at a firm charging p_i. Hence, the mean effective demand faced by the firm is $(1 - \beta)\lambda$. If I assume that the cost of serving each consumer is equal to a constant value a, I can write expected profits as a function of price as follows:[9]

$$\pi(p_i|p_i \leq B_0^*) = \lambda(p_i - a)\,[1 - \beta(p_i|p_i \leq B_0^*)],$$

or, from (2.8),

$$\pi(p_i|p_i \leq B_0^*) = \lambda(p_i - a)\,\frac{1 - \alpha z - z^{n_i^*}(1 - \alpha)}{1 - \alpha z - z^{n_i^*+1}(1 - \alpha)}, \tag{2.9}$$

with $n_i^* = [(B_1 - p_i^*)\mu]$.

To complete the derivation of the firm's profit function I now consider a firm charging $p_k \in (B_0^*, B_1^* - 1/\mu)$ (in the Appendix I show that

9. If this is interpreted as a long-run model of the firm, this amounts to an assumption of constant returns to scale with a representing the unit cost of production. If, however, one wishes to interpret this as a short-run model, then fixed costs can be subtracted from profits in (2.9), (2.14), (3.9), and (3.13) without altering any of the results (or their proofs). In the latter case this assumption is made for two reasons. First, it allows my results to be compared easily to those obtained in the literature on price dispersion (Rothschild 1973; Axel 1977; Butters 1977; Diamond 1978; or Burdett and Judd 1983). Second, this assumption seems appropriate for many service industries. It is hard to imagine why the cost of serving one customer checking out of a grocery store (or the cost of providing a cab ride to one consumer) should depend on the number of customers waiting in the queue.

$B_1^* - 1/\mu \geq B_0^*$ in equilibrium). Since $p_k > B_0^*$, consumers who do not value time always balk when arriving at firm k, and since $p_k \leq B_1^* - 1/\mu$, the maximum money price a consumer who values time will pay, there always exists a queue size small enough such that the other consumers will join. Thus, in this case, for a steady-state solution to exist we must have

$$q(x|p_k) = z(1 - \alpha)q(x - 1|p_k) \quad \text{for all } x \leq n_k^*, \tag{2.10}$$

and

$$\sum_{x=0}^{n_k^*} q(x|p_k) = 1. \tag{2.11}$$

Solving for $q(x|p_k)$, we obtain

$$q(x|p_k) = [z(1 - \alpha)]^x \frac{1 - z(1 - \alpha)}{1 - [z(1 - \alpha)]^{n_k^*+1}} \quad \text{for all } x \leq n_k^*, \tag{2.12}$$

and hence the probability of balking is given by

$$\beta\left(p_k|B_0^* < p_k \leq B_1^* - \frac{1}{\mu}\right) = \alpha + \frac{(1 - \alpha)[z(1 - \alpha)]^{n_k^*}[1 - z(1 - \alpha)]}{1 - [z(1 - \alpha)]^{n_k^*+1}}, \tag{2.13}$$

and expected profits are

$$\pi\left(p_k|B_0^* < p_k \leq B_1^* - \frac{1}{\mu}\right) = \lambda(p_k - a)\left[1 - \beta\left(p_k|B_k^* < p_k \leq B_1^* - \frac{1}{\mu}\right)\right]. \tag{2.14}$$

Market Equilibrium

A huge literature that examines the nature of equilibrium in markets with limited information already exists. Perhaps the best-known result from this literature is that if consumers know only the distribution of prices and face nonzero search costs, equilibrium is characterized by all firms charging the monopoly price.[10] Surprisingly, this result continues to hold in this model even though consumers differ in their valuation of time and in spite of the fact that they now face uncertainty in two dimensions: money price and queue length. The goal of this section is to prove this result, and it is accomplished in the following manner. First, it is shown that if firms maximize profits there can be no more than two prices charged. Next, the determination of the steady-state equilibrium values of λ and α is discussed and equilibrium is defined. The main result of this section (all firms charge the same price in equilibrium) is then presented.

10. See Rothschild (1973); Axel (1977); or Burdett and Judd (1983). For a complete description of the assumptions necessary to obtain this result, see Braverman (1980).

LEMMA 1. There are at most two profit-maximizing prices.

Proof. I claim that (2.9) and (2.14) are strictly concave in price and thus have unique maximizers. To prove the claim it is sufficient to show that $\beta(p_i)$ and $\beta(p_k)$ are strictly convex in price.[11]

$$\beta'(p_i) = \frac{-(1-\alpha)(1-z)\mu z^{n_i^*}\ell n z}{[1-\alpha z-(1-\alpha)\, z^{n_i^*+1}]^2} > 0,$$

$$\beta''(p_i) = \frac{(1-\alpha)(1-z)\mu^2(\ell n z)^2 z^{n_i^*}[1-\alpha z+(1-\alpha)z^{n_i^*+1}]}{[1-\alpha z-(1-\alpha)z^{n_i^*+1}]^3} > 0,$$

$$\beta'(p_k) = \frac{-(1-y)\mu y^{n_k^*}\ell n y}{(1-y^{n_k^*+1})^2} > 0,$$

and

$$\beta''(p_k) = \frac{y^{n_k^*}(1-y)(\ell n y)^2\mu^2(1+y^{n_k^*+1})}{(1-y^{n_k^*+1})^3} > 0,$$

with $y = z(1-\alpha)$. Q.E.D.

Let p_0 denote the price that maximizes (2.9), p_1 the price that maximizes (2.14), and r the proportion of firms that charge p_1. In addition, for convenience, the arguments that the functions β and π are conditioned on are suppressed for the remainder of the article.

I now turn to the derivation of the conditions that define the steady-state values of λ and α. The firms' arrival rate, λ, depends on the birth and death rates. Let η denote the ratio of the number of new consumers that enter the market each period to the number of firms, and let τ denote the proportion of these consumers that do not value time. Assume further that once a consumer is served she leaves the market. Then, for λ and α to be steady-state values we must have

$$\eta\tau + r\lambda\alpha = \lambda\alpha, \tag{2.15}$$

$$\eta(1-\tau) + \lambda(1-\alpha)\left[(1-r)\frac{\beta(p_0)}{1-\alpha} + r\frac{\beta(p_1)-\alpha}{1-\alpha}\right] = \lambda(1-\alpha). \tag{2.16}$$

In (2.15), $\eta\tau$ is the per-firm arrival rate of *new* customers who do not value time, and r is the proportion of *old* customers who do not value

11. In order to find equilibrium, we must be able to maximize the profit function over p. Unfortunately, this cannot be done using ordinary calculus methods since $\pi(p)$ is *not* a continuous function of p. The discontinuities are a result of the fact that n^*, the critical queue length, must be an integer, and therefore any function involving n^* is a discontinuous function of p. Since n^* appears in β, both $\beta(p)$ and $\pi(p)$ are discontinuous. Fortunately, very little is lost if we treat these functions as if they were continuous and, by doing so, we are able to avoid all of the existence problems that ensue whenever discontinuities are involved. Hence, in order to focus our full attention on the economics involved in this problem, β and π are treated as continuous functions by allowing n^* to take on noninteger values. The qualitative results that follow are not affected by this assumption.

time and balked last period. This sum gives the per-firm number of customers who do not value time and are searching at the beginning of the period. If we are in a steady state, this sum must equal the number who were searching at the beginning of the previous period, $\lambda\alpha$. Equation (2.16) is the analogous condition for consumers who value time. These equations guarantee that the arrival rate of each type of consumer remains constant over time.

We are now in a position to define equilibrium.

DEFINITION 1. For any given vector of parameters $(b, w, \eta, \tau, \mu, a)$, a steady-state market equilibrium in the presence of incomplete information is a 7-tuple $(B_0^*, B_1^*, p_1, p_0, r, \lambda, \alpha)$ where

(i) B_0^* and B_1^* are generated by the two-price distribution defined by p_1, p_0, and r (via [2.3] for $t_j = 0, 1$, respectively);
(ii) p_0 maximizes (2.9) and p_1 maximizes (2.14) given B_0^* and B_1^*;
(iii) $\pi(p_0) = \pi(p_1)$; and
(iv) λ and α satisfy (2.15) and (2.16) given p_1, p_0, r, B_0^* and B_1^*.

Condition i guarantees that B_0^* and B_1^* are the current reservation prices when consumers face a fraction r of firms charging p_1 and the remainder charging p_0. Condition ii states that firms must maximize profits, and iii guarantees that low- or high-priced firms have no incentive to switch and charge the other price. Finally, condition iv states that the number and mixture of consumers searching remains constant over time.

The remainder of this section is devoted to proving that in any steady-state market equilibrium all firms charge the same price. To do so, we make use of the following results.

LEMMA 2. In any steady-state market equilibrium with $r \in (0, 1)$ $\pi'(p_0) = 0$. That is, $p_0 < B_0^*$, so that there is no corner solution to the problem max $\pi(p_0)$ subject to $p_0 \in [a, B_0^*]$.

Proof. Since $p_1 > B_0^*$, consumers who do not value time buy only from firms charging p_0. Thus, if $t_j = 0$, (2.3) simplifies to $b = (1 - r)(B_0^* - p_0)$, which implies $B_0^* > p_0$. Q.E.D.

Lemma 2 implies that in equilibrium low-priced firms serve all consumers who do not value time and some consumers who do. They do not increase their price to B_0^* because they lose more by driving away some of those who value time than they gain by extracting more from those who do not.

For lemma 3 define

$$I(p) \equiv \frac{(1 - \alpha)(1 - z)z^{n^*}}{1 - \alpha z - (1 - \alpha)z^{n^*+1}}, \qquad (2.17)$$

$$\forall p \in \left[a, B_1^* - \frac{1}{\mu}\right], \text{ with } n^* = [(B_1^* - p)\mu],$$

$$K(p) \equiv \alpha + \frac{(1-\alpha)[z(1-\alpha)]^{n^*}[1-z(1-\alpha)]}{1-[z(1-\alpha)]^{n^*+1}}$$
$$\forall p \in \left[a, B_1^* - \frac{1}{\mu}\right], \text{ with } n^* = [(B_1^* - p)\mu]. \tag{2.18}$$

Note that $I(p)$ and $K(p)$ are just $\beta(p_i)$ and $\beta(p_k)$, respectively (see [2.8] and [2.13]), except that their domains are wider.

LEMMA 3. If $\alpha > 0$, then $K(p) > I(p) \quad \forall p \in [a, B_1^* - 1/\mu]$.

Proof. It is straightforward to show that

$$\text{sign}\,[K(p) - I(p)] = \text{sign}\,[\alpha(1-\alpha z) - H(n^*)],$$

where

$$H(n^*) \equiv z^{n^*}(1-\alpha)[1 - z + \alpha z - (1-\alpha)^{n^*}(1-z)(1-\alpha z)].$$

To complete the proof I claim that

$$\alpha(1-\alpha z) - H(n^*) > \alpha(1-\alpha z) - H(0)$$
$$= \alpha(1-\alpha z)[1 - z(1-\alpha)] > 0,$$

since $\alpha > 0$, $z < 1$.

To prove this claim it is sufficient to show (since n^* is an integer)

$$H(n^*) < H(n^* - 1).$$

But,

$$\text{sign}\,[H(n^* - 1) - H(n^*)]$$
$$= \text{sign}\,\{1 - z + \alpha z - (1-\alpha)^{n^*-1}(1-\alpha z)[1 - z(1-\alpha)]\}.$$

Since the right-hand side is increasing in n^*, it is sufficient to prove that it is positive for $n^* = 1$. For $n^* = 1$ the right-hand side reduces to $\alpha z(1 - z + \alpha z) > 0$. Q.E.D.

Lemma 3 simply states that if two firms charge identical prices and if, for some reason, consumers who do not value time refuse to buy from one firm, then the steady-state effective demand at that firm will be lower than it is at the other firm. This lemma plays a critical role in proving the main result of this section.

THEOREM 1. In any steady-state market equilibrium all firms charge the same price.

Proof. Suppose that theorem 1 is not true. Then $r \in (0, 1)$, and by lemma 2, $\pi(p_0)$ must take on the form in figure 1 (if we are in equilibrium). By lemma 3, $(p - a)\lambda[1 - I(p)] > (p - a)\lambda[1 - K(p)]$ for all $p \in [a, B_1^* - 1/\mu]$. However, the left-hand side of this expression is just $\pi(p_0)$, and the right-hand side is $\pi(p_1)$ (over the relevant ranges for p). Thus, for any p, $\pi(p_0) > \pi(p_1)$, and since low-priced firms charge the price that maximizes $\pi(p_0)$, condition iii (the equal-profit condition) cannot hold. Q.E.D.

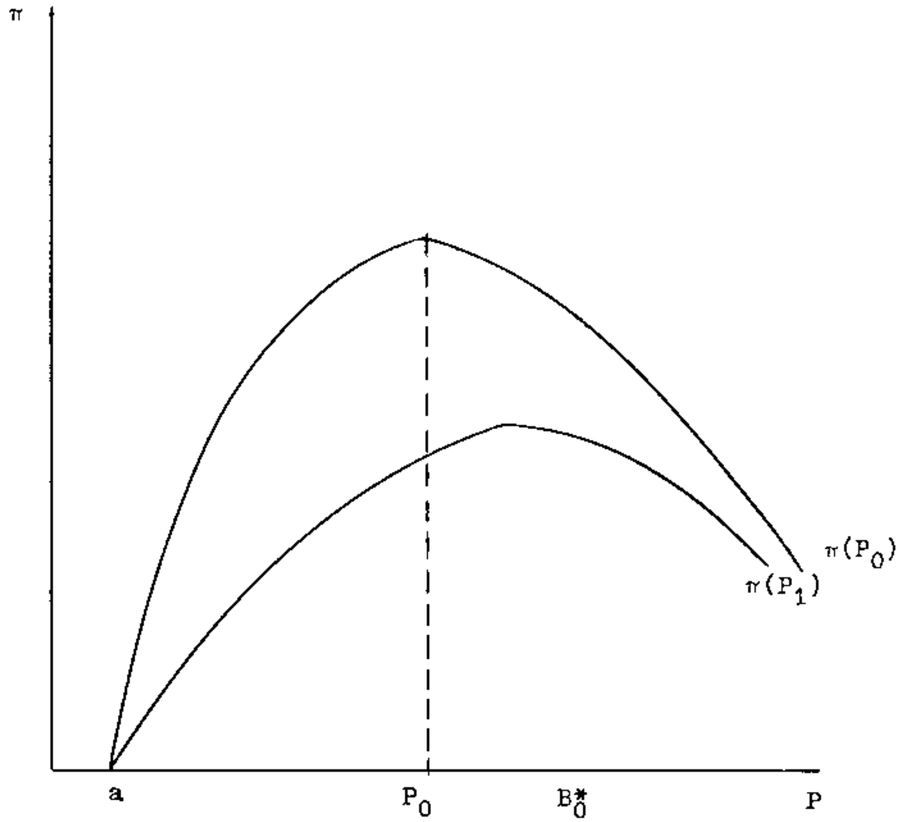

FIG. 1

In light of lemma 2, theorem 1 should not be too surprising. Since low-priced firms serve all those who do not value time *and* some who do, they always earn more profit than a high-priced firm that attempts to cater to only those who are impatient.

It is also worth noting that the nonexistence of multiple-price equilibria is a direct result of the assumption that firms are not allowed to advertise their prices. To see this, suppose that all firms are charging the same price p_0; is there any incentive for a firm to alter its price? Since p_0 maximizes $\pi(p)$ over the range $[a, B_0^*]$ (by definition) we need only consider alternative prices above B_0^*. By raising its price above B_0^* the firm effectively chooses to cater only to those consumers that value time. However, since the firm cannot advertise its price, such a price increase cannot increase the arrival rate of such customers (since they do not know how to find the firm). Although more customers who value time now join the queue (since the store is less congested), this increase in sales does not make up for the loss in demand due to losing *all* customers who do not value time. Thus, such a price increase reduces profit.

The essential point that gives us a single-price equilibrium in the standard search model continues to provide us with a single-price equilibrium in servicing industries. Since consumers do not know which firms are charging which prices, altering prices will not allow a firm to cater to a special type of consumer. Hence, there are no market forces to provide an incentive for firms to specialize in production. Such an incentive will exist with complete information (which is provided by advertisements) since any firm that lowers its price will attract all of the consumers who do not value time. Or, alternatively, any firm that increases price above B_0^* will be able to increase the arrival rate of consumers who value time.

Before moving on to consider the complete information case, I close this section by discussing some of the comparative static properties of the model. Increases in the birth rate (η) or decreases in the mean service time ($1/\mu$) holding τ fixed increase the steady-state level of congestion in the system (λ rises). In addition, since individuals who do not value time will continue to join any queue, while those who value time will balk more often (and therefore search longer), α, the steady-state proportion of consumers in the system with no cost of time, will fall. All of these effects cause B_1^* and price to rise. An increase in τ, ceteris paribus, increases the steady-state expected queue length (since fewer customers value time) and therefore increases B_1^*. In addition, since the firm now sells relatively more to those with no time cost, they have an additional incentive to raise price. Thus, increases in τ, η, or μ lead to higher prices.

III. Complete Information

In this section I assume that firms advertise their prices and then demonstrate that if they are allowed to advertise they will choose to do so. The introduction of advertisement as a control variable for the firm is not a unique feature of this article. Butters (1977) provides an interesting model in which consumers initially have no information about price, and hence firms advertise in order to solicit customers. In order to avoid the Bertrand result (i.e., price undercutting that drives price down to average cost), Butters allows firms to offer different prices to different consumers. In addition, firms need not send an advertisement to each consumer. While Butters's model accurately describes some markets,[12] it does not seem to realistically depict a number of servicing industries. Supermarkets advertise a single price for each good in newspapers that anyone is free to obtain. Service stations post gasoline prices so that any passing motorist will observe them. In some servic-

12. For example, *Time* magazine offers lower rates to students.

ing industries price discrimination of Butters's type is not possible, and any firm charges all consumers the same price.[13] Therefore, in this article, I assume that when a firm reveals its price, all consumers receive the advertisement and may purchase the good at that price. (Allowing for price discrimination would not change the qualitative nature of our results but would greatly complicate the analysis.)[14] It is argued that, in addition to simplifying the consumers' search problem, such advertising allows firms to specialize with high-priced firms serving only those who value time greatly and low-priced firms serving those with low time costs.

I retain all the assumptions of Section II except one. I now assume that consumers know which firms are charging which prices but do not know the actual queue length until they arrive at the store.

Consumer and Firm Behavior

I continue to use the same notation that was used in Section II. Let $n(p)$ denote the expected steady-state queue length at a store charging price p. Assume, once again, that there are two types of consumers: those with $t_j = 1$ and those with no cost of time. I begin the analysis by assuming that more than two prices are charged in equilibrium. Order the prices such that $p_0 < p_1 < p_2 \ldots$. *All* consumers with no cost of time will go to the stores charging p_0. If the other stores are to receive any customers at all, it must be the case that

$$p_0 + n(p_0)\frac{1}{\mu} + \frac{b + w\left(\frac{1}{\mu}\right)}{1 - \beta(p_0)} > p_1 + n(p_1)\frac{1}{\mu} + \frac{b + w\left(\frac{1}{\mu}\right)}{1 - \beta(p_1)}, \tag{3.1}$$

$$p_i + n(p_i)\frac{1}{\mu} + \frac{b + w\left(\frac{1}{\mu}\right)}{1 - \beta(p_i)} = p_j + n(p_j)\frac{1}{\mu} + \frac{b + w\left(\frac{1}{\mu}\right)}{1 - \beta(p_j)},$$
$$\text{for all } i, j \neq 0. \tag{3.2}$$

13. There are, of course, examples of servicing firms that do price discriminate in advertising. For example, firms often include coupons in advertising material that lead to different prices across customers. My analysis focuses on markets in which such discrimination is difficult or at a trivial level.

14. There are basically two types of advertisements that firms use in order to increase their profits: there are advertisements about the qualities and characteristics of the good that they are selling, which are meant to increase the demand for their product by inducing a shift in consumer's preferences, and there are price advertisements that are meant to induce consumers to buy from the firm rather than its competitors. The welfare effects of advertising designed to influence preferences are rather difficult to assess, and I make no effort to add to the already huge literature devoted to this question. The interested reader should consult Dixit and Norman (1978) for a very nice treatment of this question. I focus solely on price advertisement as a method of stealing customers from competitors.

The expected full price at i is $p_i + n(p_i)1/\mu$, and $[b + w(1/\mu)]/[1 - \beta(p_i)]$ is the expected cost of searching among firms of type i until finding one with a queue that is not full. Thus, their sum is the expected cost of obtaining the product from a firm charging p_i.

If (3.2) holds for two or more prices, then consumers with $t = 1$ will be indifferent as to which firm to visit, and hence we must make some assumption as to how the arrival rates at the firms are determined. Since consumers do not care which type of firm they end up at (as long as they do not end up at a low-priced firm), it seems logical to assume that they will choose the firm randomly, and all firms will face the same arrival rate.[15] With this assumption, if we let $1 - r$ denote the proportion of firms charging p_0 and if we assume (3.1) holds, the arrival rates will be $\lambda(1 - \alpha)/r$ for firms charging p_j, where $j \neq 0$, and $\lambda\alpha/(1 - r)$ for firms charging p_0.

For consumers with no time cost, life is very simple; they always buy from the store with the lowest price. For consumers with a nonzero cost of time, there is still an element of search involved since they do not know the actual queue length before choosing a firm. By setting the expected gain to searching one more time equal to zero, we can define the reservation price for consumers that value time. Value B_1^* must solve

$$b + w = \sum_{x=0}^{n_j^*-1} \left(B_1^* - p_j - \frac{x + 1}{\mu}\right) q(x|p_j), \tag{3.3}$$

with $n_j^* = [(B_1^* - p_j)\mu]$.

Equation (3.3) defines B_1^* given p_j, the price charged by the firm the consumer chooses.

Suppose that two firms are charging two different prices, p_j and p_i, which satisfy (3.2). Suppose firm i raises its price to $p_i + \delta$; what happens? With complete information, the market must adjust so that (3.2) still holds. To see this, suppose (3.2) did not hold at the new price. First consider the case in which

$$p_i + \delta + n(p_i + \delta)\frac{1}{\mu} + \frac{b + \frac{w}{\mu}}{1 - \beta(p_i + \delta)} < p_j + n(p_j)\frac{1}{\mu} + \frac{b + \frac{w}{\mu}}{1 - \beta(p_j)}. \tag{3.4}$$

In this case, *all* consumers who value time would shop at store i. With no one shopping at firm j, $n(p_j)$ would be zero, and the probability of

15. It might seem just as logical to assume the firm's arrival rate is proportional to its average queue length. If this assumption is used, it may be possible to generate equilibria with more than two prices and with all consumers who do not face time costs buying from the lowest-priced firm.

balking would be zero as well (since the queue would always be empty). At store i, $n(p_i + \delta)$ would be very high and $\beta(p_i + \delta)$ would be close to one since the queue would be almost always full. For small δ, this would obviously lead to a violation of (3.4). A similar argument rules out the possibility that the inequality is reversed since this would imply that firm i would lose all its customers due to an arbitrarily small price increase. Hence, when firms raise their prices the market automatically adjusts, through $n(p)$ and $\beta(p)$, to insure that (3.2) holds. In this manner, firms still possess a degree of monopoly power even when information is complete.

Now consider a firm charging p_j with $j \neq 0$. For a steady-state solution we must have

$$q(x|p_j) = \frac{z(1-\alpha)}{r}\, q(x-1|p_j), \quad \text{for all } x \le n_j^* = [(B_1^* - p_j)\mu], \tag{3.5}$$

$$\sum_{x=0}^{n_j^*} q(x|p_j) = 1. \tag{3.6}$$

Solving (3.5) and (3.6), we obtain

$$q(x|p_j) = \left[\frac{z(1-\alpha)}{r}\right]^x \left\{\frac{1 - \dfrac{z(1-\alpha)}{r}}{1 - \left(\dfrac{z(1-\alpha)}{r}\right)^{n_j^*+1}}\right\}. \tag{3.7}$$

The probability that the queue is full is given by

$$\beta(p_j) = \left(\frac{z(1-\alpha)}{r}\right)^{n_j^*} \left\{\frac{1 - \dfrac{z(1-\alpha)}{r}}{1 - \left(\dfrac{z(1-\alpha)}{r}\right)^{n_j^*+1}}\right\}. \tag{3.8}$$

Hence, for a firm charging p_j, steady-state expected profits are

$$\pi(p_j) = (p_j - a)\,\frac{\lambda(1-\alpha)}{r}\,[1 - \beta(p_j)]. \tag{3.9}$$

It is easy to show that (3.9) is strictly concave in price. Thus, (3.9) has a unique maximum, and all firms choose to charge the same price that is denoted by p_1.

For firms charging p_0, the arrival rate is $\alpha\lambda/1 - r$. For a steady-state solution we must have

$$q(x|p_0) = \frac{\alpha z}{1-r}\, q(x-1|p_0), \tag{3.10}$$

$$\sum_{x=0}^{\infty} q(x|p_0) = 1. \tag{3.11}$$

Solving (3.10) and (3.11) for $q(x|p_0)$, we obtain

$$q(x|p_0) = \left(\frac{\alpha z}{1 - r}\right)^x \frac{1}{1 - \left(\frac{\alpha z}{1 - r}\right)}. \quad (3.12)$$

Consumers with no cost of time will never balk so that

$$\pi(p_0) = (p_0 - a)\frac{\alpha\lambda}{1 - r}. \quad (3.13)$$

Equation (3.13) is maximized by setting p_0 as high as possible.

Market Equilibrium

Before describing equilibrium I must discuss the determination of the steady-state values of λ and α. With complete information, consumers who do not value time never balk, and, thus, the number searching each period is equal to the number that enter the market.

$$\eta\tau = \lambda\alpha. \quad (3.14)$$

Those that value time balk with probability $\beta(p_1)$, and, thus, for the number of such searchers to remain constant over time we must have

$$\eta(1 - \tau) + \lambda(1 - \alpha)\beta(p_1) = \lambda(1 - \alpha). \quad (3.15)$$

Equilibrium occurs when all firms maximize profits given B_1^*, λ, and α; B_1^* is generated by (3.3) given p_1 and r; $\pi(p_0) = \pi(p_1)$ so that no firm has an incentive to switch prices; and λ and α satisfy (3.14) and (3.15).

Maximizing (3.9) results in p_1, and p_0 comes from the equal profit condition (once r is determined). The determination of r is not quite as straightforward: it is a standard result in queuing theory that a steady-state solution exists if and only if the traffic intensity is less than one (if people enter the system faster than they leave, the lines get infinitely long). Thus, $1 - r$ must be large enough that $z\alpha/(1 - r) \leq 1$. In fact, if we let N denote the number of firms in the industry and N_0 the number charging p_0, equilibrium is achieved when N_0 equals the smallest integer that satisfies the condition $z\alpha/(1 - r) < 1$. To see this, consider what happens if a greater number of firms charge $p_0 < p_1$. Since low-priced firms serve only consumers who do not value time, customers never balk on arriving at low-priced stores. Such firms continue to serve customers as long as their queues are nonempty. However, if $z\alpha/(1 - r) < 1$, any firm could raise its profits by lowering its price by an arbitrary amount. This would allow the firm to attract all of the consumers that have no cost of time, thereby reducing the probability that the queue will be empty. Bertrand-like competition would continue to drive p_0 down until $p_0 = a$. However, at this point firms would have an incentive to raise price to p_1 (assuming high-priced firms earn positive profits). Hence, we cannot have $z\alpha/(1 - r) < 1$ in equilibrium. For values of r such that $z\alpha/(1 - r)$ is very close to one (the integer problem might not allow us to achieve equality), the probability that the queue is empty becomes arbitrarily close to zero, and this problem is avoided. Hence, in equilibrium, r is approximately equal to $1 - \alpha z$. The firms that charge p_0 charge the highest price possible. This is the price that equates their profits with those earned by the firms charging p_1. If p_0 rises above this level, the firms' profits will rise above $\pi(p_1)$, causing one of the firms charging p_1 to lower its price to p_0. This reduces the firms' arrival rates and competition drives price back down. Thus, to summarize, in equilibrium $r = 1 - \alpha z$, and p_0 is defined by

$$\pi(p_1) = \pi(p_0). \tag{3.16}$$

Advertising

I now turn to the question of which information structure is most likely to be observed in service industries. To answer this question, suppose that we are initially in the single-price equilibrium of Section II that results when firms are restricted from advertising. Now, expand the firms' policy space to allow for advertising; will they do so? The most likely answer to this question is yes since any firm that lowers its price by an arbitrarily small amount *and* advertises that it is doing so could attract all of the consumers who do not value time. If the firm does not advertise, it will continue to serve, on average, $1/N$th of those who do not value time and $1/N$th of those that do value time *and* join the queue ($N \equiv$ the number of firms). Thus, as long as serving all of the consumers of one type is more profitable than serving $1/N$th of all types, advertising will occur. This is clearly the case for large N.

Prices and profits as a function of b, the monetary search cost, are given in figures 2 and 3. While the figures are drawn for specific values of the parameters (i.e., $a = w = 0$, $\mu = 2$, $\eta = 1$, $\tau = .5$), the qualitative features hold for any set of values. Quantities p^* and π^* denote the equilibrium price and profit levels with no advertising, and p_1, p_0, and π denote price and profits when the prices are known. Not surprisingly, advertising leads to lower prices and profits. Thus, firms would prefer a collusive agreement restricting advertising provided the agreement is enforceable. This provides at least a partial explanation of why doctors, lawyers, dentists, and other professional groups have tried to prohibit advertising in their industries. As we have recently seen in the market for legal services, however, if advertising is allowed, it will occur since firms that advertise first can increase their profits along the transition path to the new equilibrium. Unfortunately for the firms, when the new equilibrium is reached profits are lower throughout the industry.[16]

16. This analysis assumes entry barriers keep new firms from entering the market (since the number of firms is taken as exogenously given). These entry barriers may be

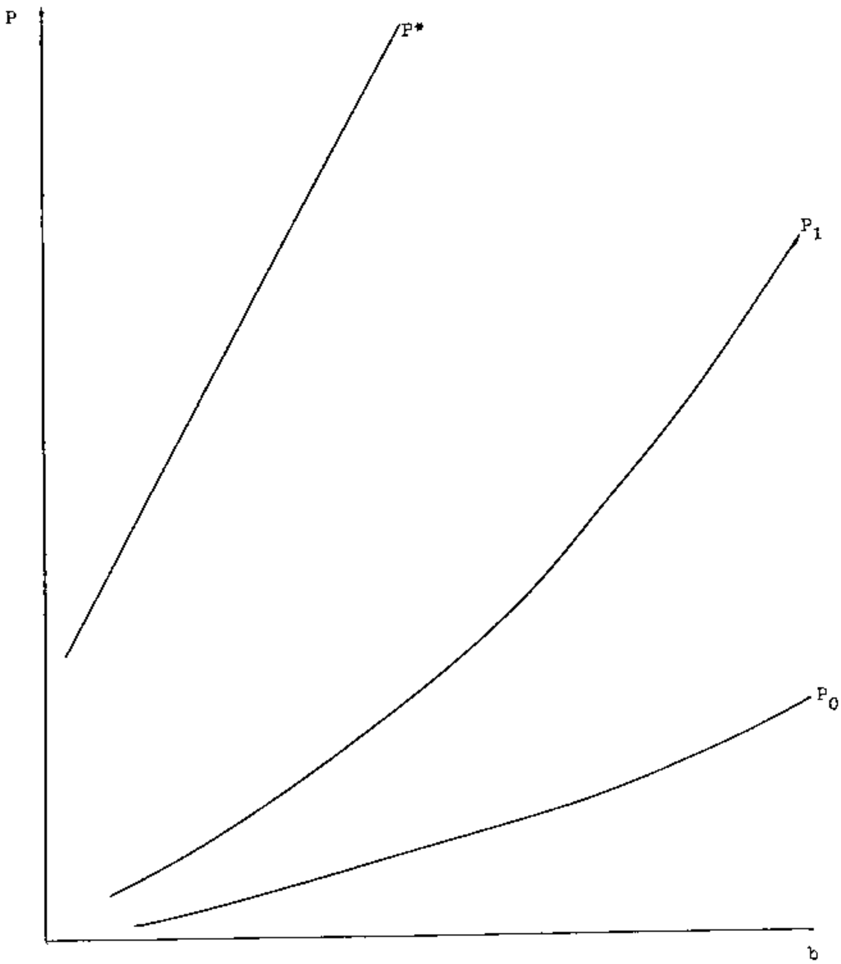

FIG. 2

IV. Conclusion

In this article a model of servicing industries was developed and used to investigate the nature of equilibrium under different information structures. I found that all firms will charge the same price if consumers do not know which firms are charging which prices and firms are not allowed to advertise. However, the market will separate efficiently if firms provide consumers with price information by advertising. Con-

due to sunk costs or licensing restrictions (as in the case of the markets for medical or legal services). With free entry the same forces would lead firms to reveal their prices, but advertising would have no effect on long-run profits (since profits would be driven to zero in either case).

FIG. 3

sumers with high time costs will pay higher prices and wait in shorter queues while consumers who face low time costs will shop at different stores, pay lower prices, and wait in longer queues. Finally, firms will advertise if allowed, but all firms would prefer an enforceable collective agreement not to advertise.

Appendix

If we assume $B_1^* - 1/\mu < B_0^*$, then for a steady-state solution we must have, for $a \leq p \leq B_1^* - 1/\mu$,

$$q(x|p) = zq(x - 1|p) \quad \text{for all } x \leq n^*, \tag{A.1}$$

$$q(x|p) = \alpha z q(x - 1|p) \quad \text{for all } x > n^*, \tag{A.2}$$

$$\sum_{x=0}^{\infty} q(x|p) = 1. \tag{A.3}$$

Solving for $q(x|p)$, we obtain

$$q(x|p) = \frac{z^x(1 - z)(1 - \alpha z)}{(1 - z^{n^*})(1 - \alpha z) + z^{n^*}(1 - z)} \quad \text{for all } x \leq n^*. \tag{A.4}$$

A customer balks at a store charging $p \leq B_1^* - 1/\mu$ if and only if he values time *and* the queue is full. Hence,

$$\beta\left(p|a \leq p \leq B_1^* - \frac{1}{\mu}\right) = \frac{(1 - \alpha)(1 - z)z^{n^*}}{(1 - z^{n^*})(1 - \alpha z) + (1 - z)z^{n^*}}. \tag{A.5}$$

For $B_1^* - 1/\mu < p < B_0^*$, only customers with no time cost join the queue so that

$$\beta\left(p|B_1^* - \frac{1}{\mu} < p \leq B_0^*\right) = 1 - \alpha.$$

Therefore,

$$\pi\left(p|a \leq p \leq B_1^* - \frac{1}{\mu}\right) = (p - a)\lambda\left[\frac{(1 - \alpha z) + z^{n^*}(\alpha - 1)}{1 - \alpha z + z^{n^*+1}(\alpha - 1)}\right], \tag{A.6}$$

$$\pi(p|B_1^* - \frac{1}{\mu} < p \leq B_0^*) = (p - a)\alpha\lambda \tag{A.7}$$

Thus, $\pi(p|B_1^* - 1/\mu < p \leq B_0^*)$ achieves a maximum at $p = B_0^*$. In order to find the maximum value of $\pi(p|a \leq p \leq B_1^* - 1/\mu)$, consider the β function

$$\beta'\left(p|a \leq p \leq B_1^* - \frac{1}{\mu}\right) = \frac{-\mu(1 - \alpha)(1 - z)z^{n^*}(\ell n z)(1 - \alpha z)}{[(1 - z\alpha) + z^{n^*+1}(\alpha - 1)]^2} > 0, \tag{A.8}$$

$$\beta''\left(p|a \leq p \leq B_1^* - \frac{1}{\mu}\right) \tag{A.9}$$

$$= \frac{z^{n^*}(1 - \alpha)(1 - z)(\ell n z)^2\mu^2(1 - z)[1 - \alpha z + z^{n^*+1}(1 - \alpha)}{[1 - \alpha z + z^{n^*+1}(\alpha - 1)]^3} > 0,$$

$$\text{for all } 0 < a < 1,\ z \neq 1.$$

Thus, $\beta(p)$ is a strictly convex function of price over the open interval $(a, B_1^* - 1/\mu)$, which implies that $\pi(p)$ is strictly concave over this interval. Hence, $\pi(p)$ has a unique maximum over this range and there exist at most two profit-maximizing prices.

Assume there is a two-price equilibrium. Let r denote the proportion of firms charging the high price B_0^*. All other firms charge a lower price, p_0. The equation defining B_0^* for a given $F(p)$, (2.3), can be rewritten as

$$B_0^* = p_0 + \frac{b}{1 - r}. \tag{A.10}$$

I am now in a position to prove the desired lemma.

LEMMA. If $b + w > \delta$ for some $\delta > 0$, no steady-state market equilibrium exists in which $B_1^* - 1/\mu \leq B_0^*$.

Proof. Since consumers that value time buy only from firms charging p_0, their expected cost of search is at least $(b + w)/(1 - r)$ (they may still balk after finding a low-priced firm). The lowest full price they could possibly pay is $p_0 + 1/\mu$. Hence, $B_1^* > p_0 + 1/\mu + (b + w)/(1 - r)$ and $B_1^* - 1/\mu > p_0 + (b + w)/(1 - r) > p_0 + b/(1 - r) = B_0^*$ by (A.10), which contradicts our initial assumption.

References

Axel, B. 1977. Search market equilibrium. *Scandinavian Journal of Economics* 79:20–40.

Braverman, A. 1980. Consumer search and alternative market equilibria. *Review of Economic Studies* 47:487–502.

Burdett, K., and Judd, K. 1983. Equilibrium price dispersion. *Econometrica* 51 (July): 955–70.

Butters, G. 1977. Equilibrium distributions of sales and advertising prices. *Review of Economic Studies* 44:465–91.

Davidson, C. 1982. Equilibrium in oligopolistic service industries. Social Science Research Institute Workshop Series Paper no. 8217. Madison: University of Wisconsin—Madison.

DeVany, A. 1976. Uncertainty, waiting time, and capacity utilization: A stochastic theory of product quality. *Journal of Political Economy* 84:523–40.

DeVany, A., and Saving, T. 1977. Product quality, uncertainty, and regulation: The trucking industry. *American Economic Review* 67:583–94.

Diamond, P. 1978. Welfare analysis of imperfect information equilibria. *Bell Journal of Economics* 9, no. 1 (Spring): 82–105.

Dixit, A., and Norman, V. 1978. Advertising and welfare. *Bell Journal of Economics* 9, no. 1 (Spring): 1–17.

Glazer, A., and Hassin, R. 1983. Search among queues. Institute for Mathematical Studies in the Social Sciences Technical Report no. 406. Stanford, Calif.: Stanford University.

Gross, P., and Harris, C. 1974. *Fundamentals of Queuing Theory*. New York: Wiley.

Haight, F. M. 1957. Queuing with balking. *Biometrika* 44 (December): 360–69.

Mann, J. K. 1974. The equilibrium of service firms. Ph.D. dissertation, University of California, Los Angeles, Department of Economics.

Riordan, M. H. 1986. Monopolistic competition with experience goods. *Quarterly Journal of Economics* 101 (May): 265–79.

Rothschild, M. 1973. Models of market organization with imperfect information: A survey. *Journal of Political Economy* 81: 1283–1308.

Shapiro, C. 1982. Consumer information, product quality and seller reputation. *Bell Journal of Economics* 13:20–35.

Shapiro, C. 1983. Premiums for high quality products as returns to reputations. *Quarterly Journal of Economics* 98:659–80.